COMPLEX EIGENVALUES OF INDEFINITE PENCIL

M. LEVITIN

For $0 < c < 2$ define the $2n \times 2n$ matrices

$$A := \begin{pmatrix} c & 1 & & & \\ 1 & c & 1 & & \\ & 1 & c & \ddots & \\ & & \ddots & \ddots & 1 \\ & & & 1 & c \end{pmatrix}, \quad B := \begin{pmatrix} 1 & & & & & \\ & \ddots & & & & \\ & & 1 & & & \\ & & & -1 & & \\ & & & & \ddots & \\ & & & & & -1 \end{pmatrix}$$

where the numbers of 1 and -1 in B coincide (equal to n). The eigenvalues of the pencil $\lambda \mapsto A - \lambda B$ are the eigenvalues of $B^{-1}A = BA$. The spectrum is symmetric with respect to both $\mathbb{R}$ and $\mathrm{i}\mathbb{R}$. The non-real eigenvalues are contained in the union of the two closed disks $\{\lambda \in \mathbb{C} : |\lambda \pm c| \leq 2\}$ whereas numerical examples suggest that they lie in their intersection.

Open problem: Prove that $|\lambda - c| \leq 2$ and $|\lambda + c| \leq 2$ holds for all $\lambda \in \sigma(BA) \setminus \mathbb{R}$.

Date: 2015 AIM.