LAPTEV-SAFRONOV CONJECTURE

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Consider a Schrödinger operator $-\Delta + V$ in $\mathbb{R}^d$ with a complex potential V.

Open problems:

i) What is the largest p such that all non-real eigenvalues lie in a disk around 0 of radius $D(\int_{\mathbb{R}^d} |V|^p \,\mathrm{d}x)^{(p-d/2)^{-1}}$ (the constant $D > 0$ shall not depend on V)?

ii) What happens to embedded eigenvalues of self-adjoint Schrödinger operators under non-self-adjoint perturbations?

Both problems are related to the Laptev-Safronov conjecture which states that for every $d \in \mathbb{N}$ and $0 < \gamma \leq d/2$ there exists a constant $D_{\gamma,d} > 0$ such that for every potential V every non-real eigenvalue λ satisfies

$$|\lambda|^\gamma \leq D_{\gamma,d} \int_{\mathbb{R}^d} |V|^{\gamma+\frac{d}{2}} \,\mathrm{d}x;$$

here $\gamma = p - d/2$ with p from problem i). The Laptev-Safronov conjecture is known to be true in dimension $d = 1$ if $\gamma = 1/2$ and in dimension $d \geq 2$ if $0 < \gamma \leq 1/2$.

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