Curved Dirichlet waveguides in strong magnetic field

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Let $\Omega \subset \mathbb{R}^2$ be a strip of constant width $2a$ built over an infinite C^4-smooth curve $\Gamma : \mathbb{R} \to \mathbb{R}^2$ which is not a straight line but it is asymptotically straight in a suitable sense; for the purpose of this note it is possible to suppose it is straight outside a compact. Suppose that Ω does not intersect itself and consider the operators $H(B) := (-i\nabla + A)^2$ in $L^2(\Omega)$ with the domain $H_0^1(\Omega) \cap H^2(\Omega)$, where A is a vector potential corresponding to the homogeneous magnetic field of intensity B perpendicular to the plane in which the strip lies.

If the magnetic field is absent, $B = 0$, the operator is the Dirichlet Laplacian and it is notoriously known that it has a non-void discrete spectrum below $\inf \sigma(H(0)) = (\frac{\pi}{2a})^2$ as one can learn in [EŠ89, DE95] and numerous subsequent papers. Exposing such a system to a *local* magnetic field stabilizes the spectrum: the essential spectrum is preserved and the strip must be sufficiently (in terms of the field) bent to produce isolated eigenvalues [EK05].

A homogeneous field is a much stronger perturbation and the question arises what happens with the spectrum under its influence. Even if Ω is straight, the essential spectrum threshold moves up if $B \neq 0$ — see, e.g. [HS08] — and under the asymptotic straightness assumption one naturally expects that $\sigma_{\text{ess}}(H(B))$ will not be affected by the curvature. For small values of $|B|$ one can use a suitable gauge [Ex93] in combination with the perturbation theory to see how the non-magnetic bound states change under influence of the field but this tells us nothing about the behaviour beyond the weak-field regime.

Conjecture: $\sigma_{\text{disc}}(H(B)) = \emptyset$ holds for $|B|$ large enough.

For mathematically minded readers I add that if the stated conjecture can be proved to be true, the effect is likely to be robust, i.e. insensitive to the regularity assumptions about Γ as long as the asymptotic straightness will guarantee preservation of the essential spectrum.

References

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