The de Sitter group representations: open questions

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Open questions/programs (see presentation of JP Gazeau)

- Complete description of the indecomposable representations involving the elements $\Pi_{p,0}$, $p = 2, 3, \dots$, in the $SO_0(1,4)$ or $Sp(2,2)$ scalar discrete series in terms of "Krein-Gupta-Bleuler multiplets" and underlying cohomology when we deal with de Sitter space-time actions.

- Building quantum field theory which is fully covariant under the UIR $\Pi_{p,0}$, $p = 2, 3, \dots$ in Dixmier notations .

- Complete description of the unitary irreducible representations $\Pi_{p,0}$, $p = 2, 3, \dots$, in the $SO_0(1,4)$ or $Sp(2,2)$ scalar discrete series when we deal with de Sitter phase space actions.

- In order to deal with (self) interaction, determine tensor product reductions

$$\Pi_{p,0} \otimes \Pi_{p',0} = \sum_{\delta \in \mathcal{UD}} \mathfrak{m}_\delta \Pi_\delta \,,$$

  where $\mathcal{UD}$ indexes the unitary dual of $SO_0(1,4)$ and $\mathfrak{m}_\delta$ is the multiplicity of Π_δ in the decomposition (difficult)

- More generally, determine tensor product reductions

$$\Pi^{\pm}_{p,q} \otimes \Pi^{\pm}_{p',q'} = \sum_{\delta \in \mathcal{UD}} \mathfrak{m}_\delta \Pi_\delta \,,$$

  (even more difficult)

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