

NON-POLYNOMIAL COMPLEX POTENTIALS

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Consider

$$H = -\frac{d^2}{dx^2} + x^m + a_1x^{m-1} + \cdots + a_m$$

in $L^2(\mathbb{R}_+)$ with $y(0) \cos \theta + y'(0) \sin \theta = 0$, where $a_j \in \mathbb{C}$ and $\theta \in \mathbb{C}$. It is known that H has infinitely many eigenvalues. Moreover, all eigenvalues are real if and only if $a_j \in \mathbb{R}$ for all j and $\theta \in \mathbb{R}$.

Open Problem: Is there any non-self-adjoint non-polynomial potential case H , either in $L^2(\mathbb{R})$ or in $L^2(\mathbb{R}_+)$, that generates infinitely many real eigenvalues and at most finitely many non-real eigenvalues?