

SCHAUDER BASIS FOR SCHRÖDINGER OPERATORS WITH PERIODIC BOUNDARY CONDITIONS

B. MITYAGIN

Consider the Schrödinger operator $-\mathrm{d}^2/\mathrm{d}x^2 + V$ on $(0, \pi)$ with periodic boundary conditions, where the potential V is a trigonometric polynomial, i.e.,

$$V(x) = \sum_{k=-m}^m v_k e^{2ikx}; \quad v_k \in \mathbb{C}, \quad |k| \leq m.$$

Open Problem: For which sets of coefficients $\{v_k\}_{k=-m}^m$ do the eigenfunctions form a Schauder basis for $L^2(0, \pi)$?

Known Case: Let $V(x) := e^{-2ix} + be^{2ix}$. Then the answer is “yes” if and only if $|b| = 1$.