ZEROS AND POLES OF NEVANLINNA FUNCTIONS

M. MARLETTA

Let $m_1, m_2, \dots$ be an infinite sequence of meromorphic functions with $\mathrm{Im}\, m_j(\lambda) > 0$ if $\mathrm{Im}\, \lambda > 0$ and $\mathrm{Im}\, m_j(\lambda) < 0$ if $\mathrm{Im}\, \lambda < 0$ (Nevanlinna functions). Suppose that there exists a non-empty interval $I \subseteq \mathbb{R}$ such that for every non-empty subinterval $J \subseteq I$ holds

$$\lim_{j\to\infty} \#\{\text{pole of } m_j \text{ in } J\} = \infty.$$

Let g be a function which is analytic in a complex neighborhood of I.

Open problems:

i) Show that for every open complex neighborhood U of J,

$$\lim_{j\to\infty} \#\{\text{zero of } (m_j - g) \text{ in } U\} = \infty.$$

ii) Show that there exists a constant $C > 0$, independent of j, such that for every open complex neighborhood U of J,

$$\big|\#\{\text{zero of } (m_j - g) \text{ in } U\} - \#\{\text{pole of } m_j \text{ in } U\}\big| \le C.$$

The result is known to hold if $\mu(J) := \lim_{j\to\infty} j^{-1}\, \#\{\text{pole of } m_j \text{ in } J\}$ exists.

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