Consider a Schrödinger operator $-\Delta + V$ in $\mathbb{R}^d$ with a complex potential $V$.

**Open problems:**

i) What is the largest $p$ such that all non-real eigenvalues lie in a disk around 0 of radius $D(\int_{\mathbb{R}^d} |V|^p \, dx)^{(p-d/2)-1}$ (the constant $D > 0$ shall not depend on $V$)?

ii) What happens to embedded eigenvalues of self-adjoint Schrödinger operators under non-self-adjoint perturbations?

Both problems are related to the Laptev-Safronov conjecture which states that for every $d \in \mathbb{N}$ and $0 < \gamma \leq d/2$ there exists a constant $D_{\gamma,d} > 0$ such that for every potential $V$ every non-real eigenvalue $\lambda$ satisfies

$$|\lambda|^\gamma \leq D_{\gamma,d} \int_{\mathbb{R}^d} |V|^{\gamma+d/2} \, dx;$$

here $\gamma = p - d/2$ with $p$ from problem i). The Laptev-Safronov conjecture is known to be true in dimension $d = 1$ if $\gamma = 1/2$ and in dimension $d \geq 2$ if $0 < \gamma \leq 1/2$. 

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