Consider a non-normal matrix $A \in \mathbb{C}^{n \times n}$. Define
\[ s(A, \varepsilon) := \inf_{\Delta, V} \|V\| \|V^{-1}\| \varepsilon + \|\Delta\|. \]

\textbf{Open problem:} Prove Davies’ conjecture (2007): There exists a constant $C_n > 0$, independent of $A \in \mathbb{C}^{n \times n}$, such that $s(A, \varepsilon) \leq C_n \sqrt{\varepsilon}$.

It is known that the conjecture holds for Jordan blocks (then $C_n = 2$ suffices) and for $3 \times 3$ matrices with $\|A\| \leq 1$ (then $C_n = 4$ suffices).