# DAVIES' CONJECTURE ABOUT APPROXIMATE DIAGONALIZATION

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Consider a non-normal matrix $A \in \mathbb{C}^{n \times n}$. Define

$$\underline{s}(A, \varepsilon) := \inf_{\substack{\Delta, V \\ V^{-1}(A+\Delta)V \text{ diagonal}}} \|V\| \, \|V^{-1}\| \varepsilon + \|\Delta\|.$$

**Open problem:** Prove Davies' conjecture (2007): There exists a constant $C_n > 0$, independent of $A \in \mathbb{C}^{n \times n}$, such that $\underline{s}(A, \varepsilon) \leq C_n \sqrt{\varepsilon}$.

It is known that the conjecture holds for Jordan blocks (then $C_n = 2$ suffices) and for $3 \times 3$ matrices with $\|A\| \leq 1$ (then $C_n = 4$ suffices).