# COMPUTATIONS OF THE INSTABILITY INDEX FOR A NON-SELF-ADJOINT OPERATORS

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The stability of steady states is a basic question about the dynamics of any partial differential equation that models the evolution of a physical system.

In order to numerically evaluate the instability index of a given differential operator $A$, its computation should be reduced to a problem of linear algebra. Particularly for problems with periodic boundary conditions, it seems natural to restrict the operator $A$ to a finite-dimensional space of trigonometric polynomials.

**Open problem:** Under what conditions the instability index (the total number of unstable eigenvalues) can be computed from the resulting finite dimensional matrix?

One difficulty is that the entries of the infinite matrix corresponding to the differential operator $A$ grow with the row and column index, so that any truncation is not a small perturbation.

If $A$ is a self-adjoint semi-bounded differential operator of even order, then the instability index can be estimated by variational methods, or computed directly from the zeros of the corresponding Evans function.

Understanding the spectrum of a non-self-adjoint operator is a much harder problem. It is not at all obvious how to restrict the computation of its instability index to a finite-dimensional subspace, or how to even estimate its dimension. Furthermore, the numerical calculation of eigenvalues can be extremely ill-conditioned even in finite dimensions.

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