SCHAUDER BASES OF PERIODIC FUNCTIONS AND MULTIPLIERS

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Let $e_n(x) := \sqrt{2}\sin(n\pi x)$. Then $\{e_n\}$ is a Schauder basis of $L^p(0,1)$ for all $p > 1$. Let $f \in C(\mathbb{R}, \mathbb{C})$ satisfy $f(x+2) = f(x)$, $f(-x) = -f(x)$, $f(1/2 + x) = f(1/2 - x)$ and define $f_n(x) := f(nx)$. Let $A : L^p(0,1) \to L^p(0,1)$ be the linear extension of the map $Ae_n = f_n$. Then $\{f_n\}$ is a Schauder basis of $L^p(0,1)$ if and only if $A : L^p(0,1) \longrightarrow L^p(0,1)$ is a bounded operator with a bounded inverse. Let $\{c_k\}$ be the Fourier coefficients of f. Then A can be written as $A = \sum_k c_k M_k$ where M_k are the linear extensions of the map $M_k e_n = e_{kn}$.

Open problem: Find necessary and sufficient conditions on $\{c_k\}$ for $0 \notin \sigma(A)$ whenever $p \neq 2$.

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