SCHAUDER BASES OF PERIODIC FUNCTIONS AND
MULTIPLIERS

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Let \( e_n(x) := \sqrt{2} \sin(n\pi x) \). Then \( \{e_n\} \) is a Schauder basis of \( L^p(0, 1) \) for all \( p > 1 \).

Let \( f \in C(\mathbb{R}, \mathbb{C}) \) satisfy \( f(x + 2) = f(x) \), \( f(-x) = -f(x) \), \( f(1/2 + x) = f(1/2 - x) \) and define \( f_n(x) := f(nx) \). Let \( A : L^p(0, 1) \to L^p(0, 1) \) be the linear extension of the map \( Ae_n = f_n \). Then \( \{f_n\} \) is a Schauder basis of \( L^p(0, 1) \) if and only if \( A : L^p(0, 1) \to L^p(0, 1) \) is a bounded operator with a bounded inverse. Let \( \{c_k\} \) be the Fourier coefficients of \( f \). Then \( A \) can be written as \( A = \sum_k c_k M_k \) where \( M_k \) are the linear extensions of the map \( M_k e_n = e_{kn} \).

**Open problem:** Find necessary and sufficient conditions on \( \{c_k\} \) for \( 0 \notin \sigma(A) \) whenever \( p \neq 2 \).