Spectral projections and resolvent bounds for quantized partially elliptic quadratic forms

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Examples and resolvent growth

We will discuss the class of elliptic(ish) quadratic operators Q. As examples, consider the complex harmonic oscillator

$$Q_\theta = -e^{-2i\theta}\frac{d^2}{dx^2} + e^{2i\theta}x^2,$$

studied by E. B. Davies and others, and quadratic Kramers-Fokker-Planck

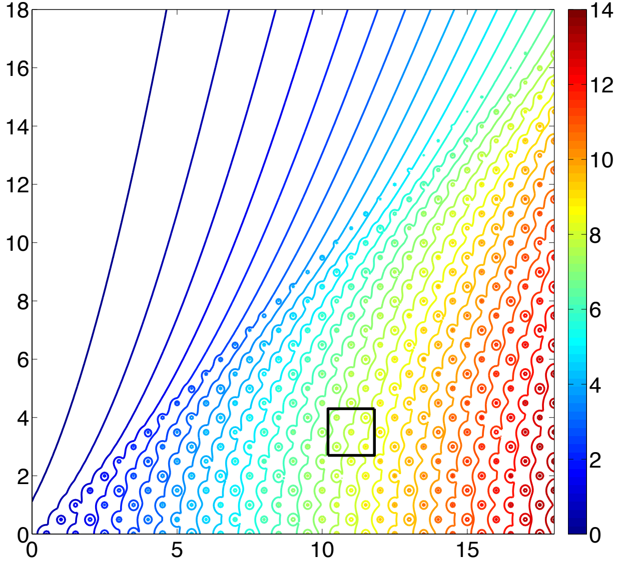
$$P_a = \frac{1}{2}(v^2 - \partial_v^2) + a(v\partial_x - x\partial_v).$$

Question: What is the behavior of the resolvent norm

$$||(Q-z)^{-1}||_{\mathcal{L}(L^2)}, \quad |z| \to \infty?$$

Particularly, what is the rate of exponential growth deep within the numerical range, close to the spectrum?

$\log_{10} ||(P_a - z)^{-1}||_{\mathcal{L}(L^2)}$ for $a = 1/\sqrt{2}$



Spectral projections

We may define spectral projections around an eigenvalue "μ_α" via

$$\Pi_\alpha = \frac{1}{2\pi i} \int_{|z-\mu_\alpha|=\varepsilon} (z-Q)^{-1}\, dz.$$

If $||\Pi_\alpha||$ large, then $||(Q-z)^{-1}||$ *somewhere* large, but not vice versa (cancellation).

We may find v_α with $||v_\alpha|| = 1$ and

$$||\Pi_\alpha|| = ||\Pi_\alpha v_\alpha||.$$

If $||\Pi_\alpha v_\alpha||$ is large, maybe so is $||(Q-z)^{-1} v_\alpha||$?

Good approximation by $||(P_a - z)^{-1} v_\alpha||$ for $a = 1/\sqrt{2}$

For corresponding α, compute relative error

$$\log_{10}\left(\frac{||(P_a - z)^{-1}||}{||(P_a - z)^{-1} v_\alpha||} - 1\right).$$