Curved Dirichlet waveguides in strong magnetic field

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Let \( \Omega \subset \mathbb{R}^2 \) be a strip of constant width \( 2a \) built over an infinite \( C^4 \)-smooth curve \( \Gamma : \mathbb{R} \to \mathbb{R}^2 \) which is not a straight line but it is asymptotically straight in a suitable sense; for the purpose of this note it is possible to suppose it is straight outside a compact. Suppose that \( \Omega \) does not intersect itself and consider the operators \( H(B) := (-i \nabla + A)^2 \) in \( L^2(\Omega) \) with the domain \( H^1_0(\Omega) \cap H^2(\Omega) \), where \( A \) is a vector potential corresponding to the homogeneous magnetic field of intensity \( B \) perpendicular to the plane in which the strip lies.

If the magnetic field is absent, \( B = 0 \), the operator is the Dirichlet Laplacian and it is notoriously known that it has a non-void discrete spectrum below \( \inf \sigma(H(0)) = (\frac{2a}{B})^2 \) as one can learn in [ES89, DE95] and numerous subsequent papers. Exposing such a system to a local magnetic field stabilizes the spectrum: the essential spectrum is preserved and the strip must be sufficiently (in terms of the field) bent to produce isolated eigenvalues [EK05].

A homogeneous field is a much stronger perturbation and the question arises what happens with the spectrum under its influence. Even if \( \Omega \) is straight, the essential spectrum threshold moves up if \( B \neq 0 \) — see, e.g. [HS08] — and under the asymptotic straightness assumption one naturally expects that \( \sigma_{\text{ess}}(H(B)) \) will not be affected by the curvature. For small values of \( |B| \) one can use a suitable gauge [Ex93] in combination with the perturbation theory to see how the non-magnetic bound states change under influence of the field but this tells us nothing about the behaviour beyond the weak-field regime.

**Conjecture:** \( \sigma_{\text{disc}}(H(B)) = \emptyset \) holds for \( |B| \) large enough.

For mathematically minded readers I add that if the stated conjecture can be proved to be true, the effect is likely to be robust, i.e. insensitive to the regularity assumptions about \( \Gamma \) as long as the asymptotic straightness will guarantee preservation of the essential spectrum.

**References**


