Matrix Hamiltonians with a chance of being complex symmetric

Question

\( \mathcal{PT} \)-symmetric matrix Hamiltonians \( H = \mathcal{P} H^\dagger \mathcal{P} \neq H^\dagger \), viz.,

\[
H^{(N)} = \begin{bmatrix}
-(N - 1) & g_1 \\
-g_1 & -(N - 3) & g_2 \\
& -g_2 & \ddots & \ddots \\
& & \ddots & N - 5 & g_2 \\
& & & -g_2 & N - 3 & g_1 \\
& & & & -g_1 & N - 1 \\
\end{bmatrix}
\] (1)

have the spectrum which only remains real and observable inside a domain \( D \) of couplings. On the boundary \( \partial D \) of stability (formed by the Kato’s exceptional points) these matrices get similar to ones containing Jordan blocks.

In some vicinity of \( \partial D \) one could try to find the form of these matrices which would be complex symmetric.

A few comments

At the dimensions \( N = 2J \) or \( N = 2J + 1 \) these models depend just on a \( J \)-plet of real couplings \( g_1, g_2, \ldots, g_J \). One of the most important formal merits of these models is that at any dimension \( N \), all the domain \( D^{(N)} \) lies inside a finite hypercube. The boundary \( \partial D^{(N)} \) itself can be characterized by its strong-coupling maxima which were obtained in the following closed form,

\[
g_n^{(\text{max})} = \pm (N - n) n, \quad n = 1, 2, \ldots, J.
\] (2)
Although the strong-coupling result (2) looks easy, its derivation required extensive computer-assisted symbolic manipulations. Via a nontrivial extrapolation guesswork we revealed that geometrically, the horizons $\partial \mathcal{D}^{(N)}$ are (hyper)surfaces with protruded spikes called extreme exceptional points, EEPs. This intuitive picture has been complemented by the more quantitative descriptions of $\partial \mathcal{D}^{(N)}$. It was based on the strong-coupling perturbation ansatz

$$g_n = g_n^{(\text{max})} \sqrt{1 - \gamma_n(t)} \quad \gamma_n(t) = t + t^2 + \ldots + t^{J-1} + G_n t^J. \quad (3)$$

using an auxiliary, formally redundant small parameter $t$.

Some references


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