Matrix Hamiltonians with a chance of being complex symmetric

Question

$\mathcal{PT}-$symmetric matrix Hamiltonians $H = \mathcal{P}H^\dagger\mathcal{P} \neq H^\dagger$, viz.,

$$H^{(N)} = \left[\begin{array}{cccccccc} -(N-1) & g_1 & & & & & \\ -g_1 & -(N-3) & g_2 & & & & \\ & -g_2 & \ddots & \ddots & & & \\ & & \ddots & N-5 & g_2 & & \\ & & & -g_2 & N-3 & g_1 \\ & & & & -g_1 & N-1 \end{array} \right] \tag{1}$$

have the spectrum which only remains real and observable inside a domain $\mathcal{D}$ of couplings. On the boundary $\partial\mathcal{D}$ of stability (formed by the Kato's exceptional points) these matrices get similar to ones containing Jordan blocks.

In some vicinity of $\partial\mathcal{D}$ one could try to find the form of these matrices which would be complex symmetric.

A few comments

At the dimensions $N = 2J$ or $N = 2J + 1$ these models depend just on a $J-$plet of real couplings $g_1, g_2, \ldots, g_J$. One of the most important formal merits of these models is that at any dimension N, all the domain $\mathcal{D}^{(N)}$ lies inside a finite hypercube. The boundary $\partial\mathcal{D}^{(N)}$ itself can be characterized by its strong-coupling maxima which were obtained in the following closed form,

$$g_n^{(max)} = \pm(N-n)\,n\,, \qquad n = 1, 2, \ldots, J\,. \tag{2}$$

Although the strong-coupling result (2) looks easy, its derivation required extensive computer-assisted symbolic manipulations. Via a nontrivial extrapolation guesswork we revealed that geometrically, the horizons $\partial\mathcal{D}^{(N)}$ are (hyper)surfaces with protruded spikes called extreme exceptional points, EEPs. This intuitive picture has been complemented by the more quantitative descriptions of $\partial\mathcal{D}^{(N)}$. It was based on the strong-coupling perturbation ansatz

$$g_n = g_n^{(max)} \sqrt{(1-\gamma_n(t))} \qquad \gamma_n(t) = t + t^2 + \ldots + t^{J-1} + G_n t^J \,. \tag{3}$$

using an auxiliary, formally redundant small parameter t.

Some references

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