Johannes Sjöstrand:

Some open problems related to the Weyl asymptotics for non-self-adjoint operators.

Consider a non-self-adjoint (pseudo)differential operator on a compact manifold; under suitable additional assumptions, including ellipticity, there are now many results (discussed in my talk) saying that if we add a small random perturbation, then with probability close to one (in the semi-classical limit) or equal to one (in the high frequency limit) the eigenvalues of the operator distribute according to Weyl's law; the number of eigenvalues in a suitable complex domain is asymptotically equal to the phase space volume of the inverse image of that domain under the principal symbol map.

One may ask many questions about the statistics of the eigenvalues. For instance what is the average separation or even the probability law for the separation of two nearest neighbors? (To give a precise meaning to the question is part of the problem.) One may also ask for the probabaility law for the number of eigenvalues in a given domain as well as the correlation or even the joint probability law for the number of eigenvalues in two different domains. One may expect that the number of eigenvalues in two disjoint domains are close to being independent random variables.

While Weyl asymptotics is a very general phenomenon which is valid for a wide class of random perturbations, the open problems above may depend very much on the class of random perturbations. It would be natural to start with the simple one dimensional model operator on the circle with Gaussian perturbations of the type studied by Hager and Bordeaux Montrieux and for which there is a very simple approach in my lecture notes from the Evian meeting in 2009 (available on arxiv, see my talk for the reference).