In 2003, Benilov, O'Brien and Sazonov proposed a model for the stability of the process used for coating fluorescent tubes. This led to a differential equation eigenproblem for which any reasonable operator realization is highly non-selfadjoint, yet for suitable parameter regimes the eigenvalues are all real. The proof of this fact is due to John Weir, Appl. Math. Lett. **22** (2009), no. 2, 280–283, where a full description of the problem may be found.

A generalization of this work (http://arxiv.org/abs/1004.2355) considers the differential equation

$$i\epsilon\frac{d}{dx}\left(f(x)\frac{du}{dx}\right)+i\frac{du}{dx}=\lambda u,$$

in which $f(x)>0$ on $(0,\pi)$, $f'(0)=1$, and f has (say) a $C^2(\mathbb{R})$ 2π-periodic odd extension to the whole real line. The parameter ϵ is real with $0<\epsilon<2$.

It is known that there exists, for each $\lambda\in\mathbb{C}$, a unique solution of the differential equation, say $\psi(x,\lambda)$, with the properties

$$\lim_{x\to 0}\psi(x,\lambda)=1,$$

$$\psi(-x,\lambda)=\psi(x,-\lambda)=\overline{\psi(x,\overline{\lambda})}.$$

Open problem: for fixed $x\in(0,\pi]$, what is the growth order of $\psi(x,\lambda)$ as an analytic function of λ?

The conjecture is that it should be $1/2$, which can be proved in many special cases, e.g. when f is linear near $x=0$.

A similar problem arises in consideration of Schrödinger equations with PT-symmetric potentials on the real line. Here one has two solutions $\psi_{left}(x,\lambda)\in L^2(-\infty,0)$ and $\psi_{right}(x,\lambda)\in L^2(0,\infty)$ – Jost solutions, for instance, if the potential lies in $L^1(\mathbb{R})$. Normalizing these solutions in some appropriate way, what can one say about their growth orders? In this case it seems that some results are available in the self-adoint case in the inverse spectral theory literature, in particular. These results all make special assumptions on the class of potential under consideration; the decay conditions are usually essential but the self-adjointness is usually not.