Decay law regularity

Let H_α be a Hamiltonian on $L^2(\mathbb{R}_+)$ with a δ interaction of strength $\alpha > 0$ at a point $a > 0$ and Dirichlet condition at the origin, that is, $H_\alpha \psi = -\psi''$ with the domain of all $\psi \in W^{2,2}(\mathbb{R}_+ \setminus \{a\})$ satisfying the boundary conditions $\psi(0) = 0$ and

$$\psi(a+) = \psi(a-) =: \psi(a)\,, \quad \psi'(a+) - \psi'(a-) = \alpha\psi(a)\,.$$

Let P be the projection onto $L^2(0,a)$ in $L^2(\mathbb{R}_+)$. Given $\psi_0 \in L^2(\mathbb{R}_+)$ such that $P\psi_0 = \psi_0$, consider the *decay law*, or survival probability at time $t > 0$,

$$P_{\psi_0}(t) := \|P\,\mathrm{e}^{-iH_\alpha t}\psi_0\|^2\,.$$

A numerical analysis of this and similar models [1] for a particular choice of ψ_0 gives a highly irregular function. A conjecture to be considered is that for $\psi_0(a-) \neq 0$ the function $P_{\psi_0}(\cdot)$ might be nowhere differentiable.

References

[1] P. Exner, M. Fraas: The decay law can have an irregular character, *J. Phys. A: Math. Theor.* **19** (2007), 1333-1340.