Open problems regarding applying non-self-adjoint operator techniques to the p-Laplace non-linear operator in one dimension

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1. Context

The q-sine functions are defined to be the eigenfunctions of a non-linear eigenvalue problem associated to the q-Laplacian subject to suitable boundary conditions on a finite interval. For $q \geq 12/11$ they are known to form a Schauder basis of the Lebesgue space $L^r(0,1)$ for all $1 < r < \infty$. The proof of this fact reduces to showing that the map T_q that sends the standard 2-Fourier sine basis into the q-sine functions extends to a linear bounded operator with a bounded inverse. Arguments involving the periodicity structure of the q-sine basis show that T_q has a non-self-adjoint lower triangular matrix representation in the 2-Fourier sine basis and that it can be expressed as a linear combination of certain isometries of $L^r(0,1)$.

2. Open problems

Even though the p-Laplacian arises naturally in applications from physics and engineering (including image processing, slow fast diffusion related to particles, superconductivity and wavelet inpainting), just a handful of rigourous results about the q-sine functions are currently known for $q \neq 2$.

Here are some open problem of current interest in this topic:

1. What are the basicity properties of the q-sine functions for $1 < q < 12/11$?
2. What are the approximation properties of this basis and its dual compared to the 2-Fourier basis for L^2 functions?
3. Is it possible to establish estimates on the decay rate on the q-sine Fourier coefficients of an L^2 function in terms of its regularity?

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4. Would it be possible to find bounds for the approximation rate in the solutions of initial and boundary value problems involving the p-Laplacian via projection methods with a q-sine or a dual q-sine basis, regarding $q > 1$ and $p > 1$ as free parameters?
5. What would be a value of q that minimise residuals for a p-Laplacian problem?

References

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