Scattering of spin waves by 1D solitons

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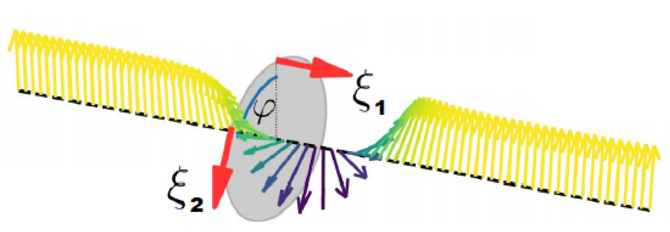
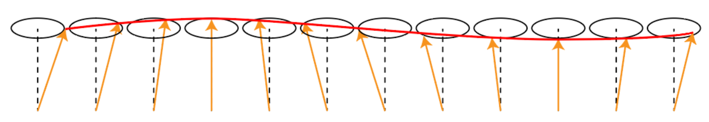
Statement of the problem

- Magnet occupying a domain $\Omega \subseteq \mathbb{R}^3$
- Magnetization: $m : \Omega \times \mathbb{R} \to \mathbb{R}^3$, $|m| = 1$.
- Soliton at equilibrium: $m = m_0$
- Perturbation (spin wave):

$$m = (1 - \xi_1^2 - \xi_2^2)^{1/2} m_0 + \xi_1 e_1 + \xi_2 e_2$$



Soliton



Spin wave

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Dynamics of spin waves

$$\partial_t \xi = \mathcal{D}\,\xi, \quad \xi : I \subseteq \mathbb{R} \to L^2(\mathbb{R}^3, \mathbb{R}^2)$$

$$\mathcal{D} = \begin{pmatrix} 0 & -\mathcal{D}_2 \\ \mathcal{D}_1 & 0 \end{pmatrix}, \qquad \xi = \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix}$$

$\mathcal{D}_1$, $\mathcal{D}_2$ are Schrödinger operators:

$$\mathcal{D}_i = -\Delta + v_i + h_i, \quad i = 1, 2$$

$$v_i \in \mathcal{S}(\mathbb{R}), \quad h_i \in \mathbb{R}, \quad h_2 > h_1 > 0$$

Fourier transform

Call $x_T = (x, y)$, $k_T = (k_x, k_y)$

Fourier transform $\xi(x_T, z)$ to $\tilde{\xi}(k_T, z)$

$$\tilde{\mathcal{D}}_i = -d^2/dz^2 + v_i(z) + h_i + k_T^2$$

Lemma $(\tilde{\mathcal{D}}_1 \tilde{\mathcal{D}}_2)^* = \tilde{\mathcal{D}}_2 \tilde{\mathcal{D}}_1$

Properties of $\tilde{\mathcal{D}}_1\tilde{\mathcal{D}}_2$, $\tilde{\mathcal{D}}_2\tilde{\mathcal{D}}_1$, and $\tilde{\mathcal{D}}$, for fixed $k_T \neq 0$

Lemma

$\sigma_p(\tilde{\mathcal{D}}_1\tilde{\mathcal{D}}_2) = \sigma_p(\tilde{\mathcal{D}}_2\tilde{\mathcal{D}}_1)$, $\sigma_c(\tilde{\mathcal{D}}_1\tilde{\mathcal{D}}_2) = \sigma_c(\tilde{\mathcal{D}}_2\tilde{\mathcal{D}}_1) = [r_0(k_T), \infty)$, $r_0(k_T) > 0$.

Lemma $\lambda \in \sigma_p(\tilde{\mathcal{D}})$ iff $\lambda^2 \in \sigma_p(\tilde{\mathcal{D}}_1\tilde{\mathcal{D}}_2)$ and $\lambda \in \sigma_c(\tilde{\mathcal{D}})$ iff $\lambda^2 \in \sigma_c(\tilde{\mathcal{D}}_1\tilde{\mathcal{D}}_2)$

For $\lambda \in \sigma_c(\tilde{\mathcal{D}}_1\tilde{\mathcal{D}}_2)$ the ODE $(\tilde{\mathcal{D}}_1\tilde{\mathcal{D}}_2 - \lambda)u = 0$ has two bounded l.i. solutions, $\chi_j(z, \lambda, k_T)$, $j = 1, 2$. From them, spectral resolution for $\tilde{\mathcal{D}}_1\tilde{\mathcal{D}}_2$ (Kemp 1960).

Spectral resolution for $\mathcal{D}$

Theorem Assume that, for fixed k_T, $\sigma_p(\tilde{\mathcal{D}}_1\tilde{\mathcal{D}}_2)$ is finite and $\sigma_p(\tilde{\mathcal{D}}_1\tilde{\mathcal{D}}_2) \cap \overline{\sigma_c(\tilde{\mathcal{D}}_1\tilde{\mathcal{D}}_2)} = \emptyset$. Let $\xi \in \mathcal{S}(\mathbb{R}^3) \times \mathcal{S}(\mathbb{R}^3)$. There are rapidly decreasing continuous functions $b_{is}(k_T)$ and $c_{js}(\lambda, k_T)$ such that

$$\xi(x_T, z) = \int_{\mathbb{R}^2} \mathrm{e}^{\mathrm{i}k_T \cdot x_T} \sum_{s=\pm 1} \left[\sum_{i=1}^{n} b_{is}(k_T) \xi_{\mathrm{P}}^{(is)}(z, k_T) + \sum_{j=1}^{2} \int_{r_0(k_T)}^{\infty} c_{js}(\lambda, k_T)\, \xi^{(js)}(z, \lambda, k_T)\, d\lambda \right] \frac{dk_T}{2\pi}, \quad \xi^{(j,s)} = \begin{pmatrix} \frac{1}{\mathrm{i}\sqrt{\lambda}} \tilde{\mathcal{D}}_2 \chi_j \\ s\,\chi_j \end{pmatrix}$$

where, for fixed k_T, $\xi_{\mathrm{P}}^{(is)}(z, k_T)$ is built from the eigenfunctions of $\tilde{\mathcal{D}}_1\tilde{\mathcal{D}}_2$.

Solution of the spin wave equation

Corollary Assume that, for fixed k_T, $\sigma_p(\tilde{\mathcal{D}}_1\tilde{\mathcal{D}}_2)$ is finite and $\sigma_p(\tilde{\mathcal{D}}_1\tilde{\mathcal{D}}_2) \cap \overline{\sigma_c(\tilde{\mathcal{D}}_1\tilde{\mathcal{D}}_2)} = \emptyset$. Let $\eta \in \mathcal{S}(\mathbb{R}^3) \times \mathcal{S}(\mathbb{R}^3)$. Then

$$\xi(x_T, z, t) = \int_{\mathbb{R}^2} \mathrm{e}^{\mathrm{i}k_T \cdot x_T} \left[\sum_{is} \mathrm{e}^{-\mathrm{i}s\sqrt{\lambda_i}(t - t_0)} b_{is}(k_T) \xi_{\mathrm{P}}^{(is)}(z, k_T) + \sum_{js} \int_{r_0(k_T)}^{\infty} \mathrm{e}^{-\mathrm{i}s\sqrt{\lambda}(t - t_0)} c_{js}(\lambda, k_T)\, \xi^{(js)}(z, \lambda, k_T)\, d\lambda \right] \frac{dk_T}{2\pi}$$

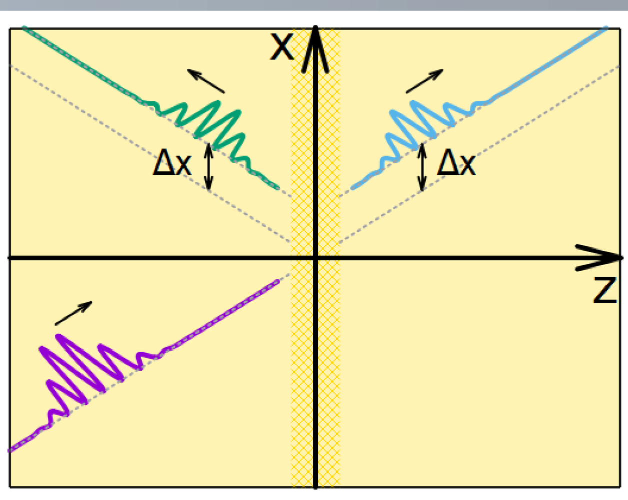
where $\lambda_i(k_T)$, $i = 1, \ldots, n$ are the eigenvalues of $\tilde{\mathcal{D}}_1\tilde{\mathcal{D}}_2$, is a solution of $\partial_t \xi = \mathcal{D}\xi$ with $\xi(x_T, z, t_0) = \eta(x_T, z)$.

Application: scattering of spin waves by 1D solitons

If $v_i(z)$ are even functions (solitons), then for $z \to \pm\infty$

$$\chi_1(z, \lambda, k_T) = \cos(kz \pm \delta_0) + o(1)$$

$$\chi_2(z, \lambda, k_T) = \sin(kz \pm \delta_1) + o(1)$$



Scattering properties can be obtained from $\mathcal{D}_1\mathcal{D}_2$

$\sigma = \cos(\delta_0 - \delta_1)\, \mathrm{e}^{\mathrm{i}(\delta_0 + \delta_1)}$ (Transmission amplitude)

$\rho = \sin(\delta_0 - \delta_1)\, \mathrm{e}^{\mathrm{i}(\delta_0 + \delta_1 + \pi/2)}$ (Reflection amplitude)

$R = \sin^2(\delta_0 - \delta_1)$ (Reflection coefficient)

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