

NSA AT CIRM  
MARSEILLE, APRIL 20–24, 2026

## Schedule and Abstracts

MONDAY, APRIL 20 (COST mSPACE SESSION IN THE AFTERNOON)

08.50 - 09:00	<i>Welcome &amp; Opening</i>	
09.00 - 09.40	<b>Beatrice Pelloni:</b> <i>Jumps, cusps and fractals in the solution of dispersive equations</i>	chair: H. Cornean
09.45 - 10.10	<b>Tom ter Elst:</b> <i>Kato's inequality and degenerate elliptic operators</i>	
10.15 - 10.45	<i>Coffee break</i>	
10.45 - 11.25	<b>Julien Royer:</b> <i>Energy decay for the wave equation with unbounded damping</i>	chair: L. Roncal
11.30 - 12.10	<b>Martin Vogel:</b> <i>Weyl law for the exponentially small singular values of the <math>d</math>-bar operator</i>	
12.30 - 14.30	<i>Lunch</i>	
14.30 - 14.40	<b>Delio Mugnolo:</b> <i>COST Action CA24122 mSpace</i>	
14.45 - 15.10	<b>Maryna Kachanovska:</b> <i>Wave Propagation in a Waveguide Filled with a Cold Strongly Magnetized Plasma</i>	chair: D. Mugnolo
15.15 - 15.40	<b>Laura Baldelli:</b> <i>Schrödinger operators in electromagnetic fields</i>	
15.45 - 16.10	<b>Cristian Cazacu:</b> <i>Hardy-Rellich inequalities with distance function to the boundary and some applications</i>	
16.15 - 16.30	<i>Coffee break</i>	
16.30 - 18.30	<b>Posters</b>	
<p><b>C. Ana Maria Adelina:</b> <i>Improved <math>H^2</math> Kato regularity for the Laplacian with relatively bounded perturbations</i></p> <p><b>M. Averseng:</b> <i>Quantum tunneling through complex-valued potentials</i></p> <p><b>J. Barnett:</b> <i>Exceptional Flow</i></p> <p><b>U. Das:</b> <i>Behavior of the ground state energy under weak perturbation of critical quasilinear operators on <math>\mathbb{R}^N</math></i></p> <p><b>C. Drysdale:</b> <i>Computation and Verification of Spectra for Non-Hermitian Systems</i></p> <p><b>P. Ebnicher:</b> <i>Neumann Realisation of Schrödinger Operators with Unbounded Complex Potentials</i></p> <p><b>M. Fakhfakh:</b> <i>Generalized weakly demicompact linear relations and their spectral properties</i></p> <p><b>F. Fischer:</b> <i>Characterizations of <math>p</math>-Parabolicity on Graphs</i></p> <p><b>N. Frantz:</b> <i>Spectral analysis of a semi-classical Bloch-Torrey operator</i></p> <p><b>J. Havel:</b> <i>Local density of states for 2D-Schrödinger operators with <math>\delta</math>-point interactions</i></p> <p><b>M. Kučera:</b> <i>Spectrum of the wave equation with Dirac damping on a compact star graph</i></p> <p><b>V. Louatron:</b> <i>Resonances for matrix-valued Schrodinger operators via normal forms</i></p> <p><b>D. Machado:</b> <i>Limiting absorption principle for a waveguide filled with a hyperbolic medium</i></p> <p><b>F. Moscatelli:</b> <i>Mode stability of self-similar wave maps outside of corotational symmetry</i></p> <p><b>R. Sannipoli:</b> <i>Geometrical bounds for the Torsion with Robin boundary conditions</i></p> <p><b>P. Schlosser:</b> <i>The Neumann trace for unbounded Lipschitz domains</i></p> <p><b>S. K. Soni:</b> <i>Extension theory for symmetric positive systems of first order PDEs</i></p> <p><b>E. Stefanescu:</b> <i>Random Schrödinger operators on compact manifolds</i></p> <p><b>M. Tušek:</b> <i>Continuum limit for discrete block operator matrices</i></p> <p><b>É. Vacelet:</b> <i>The superadiabatic projectors method applied to the spectral theory of magnetic operators</i></p> <p><b>J. Waclawek:</b> <i>Optimal discrete Hardy-Rellich-Birman inequalities</i></p>		
19.30	<i>Dinner</i>	

TUESDAY, APRIL 21 (COST mSPACE SESSION IN THE AFTERNOON)

08.50 - 09.30	<b>Claude Warnick:</b> <i>Non-self-adjoint operators and black hole ring-down</i>	chair: B. Pelloni
09.35 - 10.00	<b>Perry Kleinhenz:</b> <i>Geometry of wave damping on the torus</i>	
10.05 - 10.30	<b>Irfan Glogić:</b> <i>On spectral stability of self-similar solutions to nonlinear wave equations</i>	
10.35 - 11.05	<i>Coffee break</i>	
11.05 - 11.45	<b>Zoe Wyatt:</b> <i>Stability for relativistic fluids on slowly expanding cosmological spacetimes</i>	chair: H. Mizutani
11.50 - 12.15	<b>Gabriel Cardoso:</b> <i>An integrable structure underlying the classical double copy</i>	
12.30 - 14.30	<i>Lunch</i>	
14.30 - 15.10	<b>Jose Luis Jaramillo:</b> <i>Non-selfadjoint operators in gravity: two case studies</i>	chair: L. Ignat
15.15 - 15.40	<b>Nicolas Weber:</b> <i>Operator families in a weak coupling setting</i>	
15.45 - 16.15	<i>Coffee break</i>	
16.15 - 18.00	<b>Open Problems</b>	chair: D. Krejčířík
19.30	<i>Dinner</i>	

WEDNESDAY, APRIL 22

08.50 - 09.30	<b>Haruya Mizutani:</b> <i>Large data modified wave operators for the 1D defocusing cubic NLS</i>	chair: M. Vogel
09.35 - 10.00	<b>Yehuda Pinchover:</b> <i>Strictly positive optimal Hardy weight via Green potential</i>	
10.05 - 10.30	<b>Christiane Tretter :</b> <i>Essential numerical ranges</i>	
10.35 - 11.05	<i>Coffee break</i>	
11.05 - 11.45	<b>Horia Cornean:</b> <i>A spectral and dynamical story related to the classical Euler-Bernoulli polynomial approximation</i>	chair: S. Rota Nodari
11.50 - 12.15	<b>Martin Halla:</b> <i>Analysis of circular waveguides</i>	
12.30 - 14.30	<i>Lunch</i>	
14.30 - 19.30	<i>Free afternoon</i>	
19.30	<i>Dinner</i>	

THURSDAY, APRIL 23

08.50 - 09.30	<b>San Vu Ngoc:</b> <i>Exponentially sharp spectrum of non-selfadjoint 1D operators via analytic microlocal analysis</i>	chair: C. Warnick
09.35 - 10.00	<b>František Štampach:</b> <i>The spectral pair for Schrödinger operators with complex integrable potentials</i>	
10.05 - 10.30	<b>Denis Grebenkov:</b> <i>Mixed Steklov-Neumann-Dirichlet spectral problems: asymptotic analysis and applications</i>	
10.35 - 11.05	<i>Coffee break</i>	
11.05 - 11.45	<b>Simona Rota Nodari:</b> <i>Stability Analysis of Quantum Dissipative Systems</i>	chair: Z. Wyatt
11.50 - 12.15	<b>Anton Arnold:</b> <i>Short- and long-time behavior in evolution equations: the role of the hypocoercivity index</i>	
12.30 - 14.30	<i>Lunch</i>	
14.30 - 15.10	<b>Matthieu Léautaud:</b> <i>Small-Time Parametrix for the Fokker-Planck Equation and Applications</i>	chair: J. L. Jaramillo
15.15 - 15.40	<b>Martin Grothaus:</b> <i>Weak hypocoercivity for operator semigroups with non-selfadjoint generator</i>	
15.45 - 16.15	<i>Coffee break</i>	
16.15 - 18.30	<b>Open Problems</b>	chair: L. Cossetti
19.30	<i>Festive dinner</i>	

FRIDAY, APRIL 24

08.50 - 09.30	<b>Luz Roncal:</b> <i>Uniqueness for discrete equations: a Landis-type perspective</i>	chair: J. Royer
09.35 - 10.00	<b>Rubén de la Fuente Fernández:</b> <i>An optimal fractional Hardy inequality on the discrete half-line</i>	
10.05 - 10.30	<b>Nico Michele Schiavone:</b> <i>Uniform resolvent estimates for the Heisenberg sublaplacian</i>	
10.35 - 11.05	<i>Coffee break</i>	
12.30 - 14.30	<i>Lunch</i>	

# Abstracts

## INVITED TALKS

### 1. Horia Cornean

Title: *A spectral and dynamical story related to the classical Euler-Bernoulli polynomial approximation*

Abstract: This talk will try to remain at an elementary technical level while its main purpose is to build a bridge between hard-core people working on spectral theory of (non)self-adjoint operators, and hard-core people working on discrete dynamical systems. To be more precise, for  $\beta > 1$  we consider a discrete dynamical system induced by the map  $T : [0, 1) \rightarrow [0, 1)$  where  $T(x) = \beta x - \lfloor \beta x \rfloor$  and we investigate some spectral and dynamical (ergodic) properties of its associated (non-selfadjoint) Perron-Frobenius operator. Also, when  $\beta \geq 2$  is an integer, we establish an unexpected connection to the classical Euler-Bernoulli approximation formula.

The talk is based on joint works with G. Marcelli and I. Herbst (to appear in J. Spec. Th. <https://arxiv.org/abs/2502.06511>) and with K. Sørensen (<https://www.sciencedirect.com/science/article/pii/S0021904525000929>)

### 2. Jose Luis Jaramillo

Title: *Non-selfadjoint operators in gravity: two case studies*

Abstract: In this talk we present two physical problems, framed in a gravitational setting, defined in terms of non-selfadjoint operators. Specifically, such problems address spectral and dynamical properties of black holes described in general relativity. The first one focuses on a notion of stability for the apparent horizon of a black hole, controlled by the spectrum of the so-called MOTS-stability operator, a non-selfadjoint operator in the rotating case. The characterisation of the qualitative properties of its spectrum is of relevance in different black hole settings. The second problem concerns the linear dynamical regime in the evolution of black holes close to equilibrium. Adopting a hyperboloidal spacetime slicing description to enforce outgoing boundary conditions, the infinitesimal time generator is cast as a non-selfadjoint operator. This feature results in the spectral instability of the so-called quasi-normal mode frequencies, a potential issue for the "black hole spectroscopy programme" in gravitational wave physics. Beyond these spectral aspects, non-modal dynamical effects as growth transients or pseudo-resonances are also the subject of ongoing research in the merger of black holes. These examples provide two open problems in gravitational physics where the input of expertise in non-selfadjoint operator theory is crucial. Finally, as a sort of disclaimer, our emphasis in the physics somehow reverses the spirit in the conference title, in what could be rather paraphrased as "physical aspects of the mathematics with non-self-adjoint operators".

### 3. Matthieu Léautaud

Title: *Small-Time Parametrix for the Fokker-Planck Equation and Applications*

Abstract: The Fokker-Planck equation is a partially dissipative, non-self-adjoint equation coming from kinetic theory. We construct an operator that solves this equation approximately (a parametrix) in small time. As applications, we prove subelliptic inequalities and spectral asymptotic estimates for the Fokker-Planck stationary operator.

This is joint work with Paul Alphonse, Jean-Marc Bouclet, and Xue-Ping Wang.

### 4. Haruya Mizutani

Title: *Large data modified wave operators for the 1D defocusing cubic NLS*

Abstract: This talk concerns scattering theory within the context of final state problems for the defocusing cubic nonlinear Schrödinger equation in one space dimension. The aim is to construct a unique global solution that scatters at temporal infinity to a given prescribed asymptotic profile with a nonlinear phase modification compared to the free solution. The cubic nonlinearity in 1D is critical because there are no nontrivial solutions that scatter to the free solution. However, scattering to a modified free solution (modified scattering) has been well studied in the small data case. I will present a recent result on modified scattering for the large data case. The proof is based on a linearization around the asymptotic profile, which leads to the study of a pseudo-conformal type energy estimate for a linear Schrödinger equation with time-dependent, complex-valued and long-range potentials.

This is based on joint work with Masaki Kawamoto (Okayama University).

### 5. Beatrice Pelloni

Title: *Jumps, cusps and fractals in the solution of dispersive equations*

Abstract: I will discuss the unexpected changes in regularity in the behaviour of periodic solutions of dispersive equations, through the asymptotic study of their spectral structure.

### 6. Luz Roncal

Title: *Uniqueness for discrete equations: a Landis-type perspective*

Abstract: In this talk, we discuss recent Landis-type uniqueness results for the semidiscrete heat equation and for stationary discrete Schrödinger equations. Using quantitative estimates on spatial lattices, we identify an interpolation phenomenon linking discrete and continuum regimes. In particular, in the elliptic case, the decay exponent obtained near the continuum limit coincides with that appearing in the classical Euclidean theory involving complex-valued potentials.

Joint work with Aingeru Fernández-Bertolin and Diana Stan.

7. **Simona Rota Nodari**

Title: *Stability Analysis of Quantum Dissipative Systems*

Abstract: In this talk I will present the stability properties of plane wave solutions for a system describing quantum particles interacting with a complex environment. From a mathematical point of view, this amounts to studying a system of PDEs coupled in a non-local (in time and space) way, which complicates the analysis considerably compared to the usual nonlinear Schrödinger equations. The strategy adopted is based on the identification of suitable Hamiltonian structures and Lyapunov functionals. Work in collaboration with T. Goudon.

8. **Julien Royer**

Title: *Energy decay for the wave equation with unbounded damping*

Abstract: In this talk, we discuss the long-time behavior of the wave equation with potentially singular and unbounded damping, with a focus on the role of low frequencies. We establish resolvent estimates near zero and derive sharp polynomial decay rates for the wave and its energy in this general framework. This is a joint work with A. Arnal, B. Gerhat and P. Siegl.

9. **Martin Vogel**

Title: *Weyl law for the exponentially small singular values of the d-bar operator*

Abstract: We study the exponentially small singular values of the semiclassical d-bar operator on an exponentially weighted  $L^2$  space on a compact Riemann surface. We will assume that the Laplacian of the exponential weight changes sign along a curve. We will introduce the notion of upper and lower bound weights which give together with the orthogonal Bergman projection precise upper and lower bounds on the number of small singular values. Solving a free boundary value problem we obtain optimal weights which yield Weyl asymptotics for the counting function of exponentially small singular values. We also provide a precise description of the leading term of the Weyl asymptotics in the regime of small exponential decay rates of the singular values.

This talk is based on joint work with J. Sjöstrand and M. Hitrik.

10. **San Vu Ngoc**

Title: *Exponentially sharp spectrum of non-selfadjoint 1D operators via analytic microlocal analysis*

Abstract: I will report on recent advances concerning the computation of spectra of semiclassical 1D (pseudo)differential or Toeplitz operators with exponentially small errors. The general idea is to use a bit of geometry to analyse classical Hamiltonians in the complexified phase space. At the quantum level, this leads to complex Fourier integral operators via Sjöstrand's theory. I will present applications to non-selfadjoint perturbations of self-adjoint operators, and open questions related to the non-perturbative regime. A large part of the talk will be based on results by Durauffour and Reguer.

11. **Claude Warnick**

Title: *Non-self-adjoint operators and black hole ring-down*

Abstract: It has been an empirical fact since the 60s that a perturbed black hole returns to its ground state by producing radiation at a discrete set of complex frequencies. While this strongly suggests the spectrum of some non-self-adjoint operator underlies this phenomenon, a clean understanding of how this comes about has only been established fairly recently. I will review results obtained so far and discuss open directions.

12. **Zoe Wyatt**

Title: *Stability for relativistic fluids on slowly expanding cosmological spacetimes*

Abstract: On a background Minkowski spacetime, the Euler equations (both relativistic and not) are known to admit unstable homogeneous solutions with finite-time shock formation. Such shock formation can be suppressed on cosmological spacetimes whose spatial slices expand at an accelerated rate. However, situations with decelerated expansion, which are relevant in our early universe, are not as well understood. I will present some recent joint work in this direction, based on collaborations with David Fajman, Maciej Maliborski, Todd Oliynyk and Max Ofner.

### 1. Anton Arnold

Title: *Short- and long-time behavior in evolution equations: the role of the hypocoercivity index*

Abstract: The "index of hypocoercivity" is defined via a coercivity-type estimate for the self-adjoint/skew-adjoint parts of the generator, and it quantifies 'how degenerate' a hypocoercive evolution equation is, both for ODEs and for evolutions equations in a Hilbert space. We show that this index characterizes the polynomial decay of the propagator norm for short time and illustrate these concepts for the Lorentz kinetic equation on a torus.

Discrete time analogues of the above systems (obtained via the mid-point rule) are contractive, but typically not strictly contractive. For this setting we introduce "hypocontractivity" and an "index of hypocontractivity" and discuss their close connection to the continuous time evolution equations.

This talk is based on joint work with F. Achleitner, E. Carlen, E. Nigsch, and V. Mehrmann.

References:

- [1] F. Achleitner, A. Arnold, E. Carlen, The Hypocoercivity Index for the short time behavior of linear time-invariant ODE systems, J. of Differential Equations (2023).
- [2] A. Arnold, B. Signorello, Optimal non-symmetric Fokker-Planck equation for the convergence to a given equilibrium, Kinetic and Related Models (2022).
- [3] F. Achleitner, A. Arnold, V. Mehrmann, E. Nigsch, Hypocoercivity in Hilbert spaces, J. of Functional Analysis (2025).

### 2. Laura Baldelli

Title: *Schrödinger operators in electromagnetic fields*

Abstract: In quantum mechanics, Schrödinger operators with electric and magnetic fields model the energy of a non-relativistic particle subject to electromagnetic forces. In particular, magnetic fields add significant complexity, as their contribution via the vector potential results as imaginary perturbations of the Laplacian. This requires working in complex-valued functional spaces and often limits the applicability of standard analytical methods.

In recent joint work with Roberta Filippucci (University of Perugia, Italy) and David Krejčířík (Czech Technical University in Prague, Czech Republic), we analyze a critical quasilinear Schrödinger equation involving the magnetic  $p$ -Laplacian  $\Delta_{A,p}$  and a critical nonlinearity under suitable assumptions.

### 3. Gabriel Cardoso

Title: *An integrable structure underlying the classical double copy*

Abstract: The classical double copy is a correspondence between gravity and Yang–Mills theory, extending the so-called double copy concept from scattering amplitudes in quantum field theory to classical solutions of the field equations. This relationship was first explored in the context of spacetime metrics that can be written in Kerr–Schild form.

In this talk, we present a different approach to the classical double copy, based on the underlying integrable structure shared by both Einstein's equations and self-dual Yang-Mills equations. We focus on a subspace of solutions that is associated to an integrable system in two dimensions and which can be obtained by Wiener–Hopf factorisation of monodromy matrices.

Based on ongoing work with Cristina Câmara.

### 4. Cristian Cazacu

Title: *Hardy-Rellich inequalities with distance function to the boundary and some applications*

Abstract: We present generalizations to the  $L^p$ -setting of the Hardy-Rellich inequalities on domains of  $\mathbb{R}^N$  with singularities given by the distance function to the boundary. We obtain either sharp constants in bounded domains (where we provide concrete minimizing sequences) or find new bounds for the sharp constant, while also depending on the geometric properties of the domain and its boundary. We also give applications to the existence and non-existence of solutions for a  $p$ -Biharmonic singular problem.

This talk is based on a paper accepted for publication to SIAM. J. Math. Anal., written in collaboration with Teodor Rugina (PhD Student, University of Bucharest. Email: teorugina@yahoo.com).

### 5. Tom ter Elst

Title: *Kato's inequality and degenerate elliptic operators*

Abstract: We consider a second-order divergence form operator with real bounded coefficients and maximal domain in  $L_p(\mathbb{R}^d)$ . We assume that the matrix of principal coefficients is positive semi-definite.

Under suitable differentiability conditions on the coefficients we show that the operator is the (minus) generator of a  $C_0$ -semigroup if  $p$  is finite, and if in addition  $p > 1$ , then the space of test-functions is a core for the maximal operator. We also discuss perturbation of the maximal operator with a positive potential and optimal results in one dimension, that is if  $d = 1$ .

This talk is based on joint work with Wolfgang Arendt and with Tan Do.

### 6. Rubén de la Fuente Fernández

Title: *An optimal fractional Hardy inequality on the discrete half-line*

Abstract: Hardy type inequalities have been a widely studied field since the first proofs in the early 20th century, first by Hardy but also due to the contributions of many mathematicians like Landau, Pólya, Schur, or Riesz among others. The interest in these inequalities comes from their many applications in functional analysis, PDEs, spectral theory, or probability. In this context, we are interested in a discrete fractional Hardy's inequality, in particular a Hardy inequality concerning the fractional Laplacian in the discrete half-line.

We provide a Hardy weight that is optimal, in the sense that it cannot be substituted by any pointwise larger weight and its decay is as slow as possible. The strategy of our proof relies mainly on spectral properties of the fractional Laplacian over the natural numbers derived in [2] and the criticality theory for graph Laplacians developed in [4], with some modifications tailored for non-locally finite graphs inspired by the approach in [1] and [3]. In particular, our result answers the question posed in [2] about finding the best constant for a particular Hardy weight, and it gives an optimal weight for the classical Hardy's inequality, alternative to the one given in [5]. As an immediate application, we derive some unique continuation results for positive Schrödinger operators involving the fractional Laplacian on the discrete half-line. Joint work with Ujjal Das.

References:

- [1] Óscar Ciaurri and Luz Roncal. *Hardy's inequality for the fractional powers of a discrete Laplacian*. The Journal of Analysis, 26, 11 2018.
- [2] Borbala Gerhat, David Krejčířík, and František Štampach. *Criticality transition for positive powers of the discrete Laplacian on the half line*. Revista Matemática Iberoamericana, 41, 01 2025.
- [3] Matthias Keller and Marius Nietschmann. *Optimal Hardy inequality for fractional Laplacians on the integers*. Annales Henri Poincaré, 24(8):2729–2741, April 2023.
- [4] Matthias Keller, Yehuda Pinchover, and Felix Pogorzelski. *Criticality theory for Schrödinger operators on graphs*. Journal of Spectral Theory, 10, 08 2017.
- [5] Matthias Keller, Yehuda Pinchover, and Felix Pogorzelski. *Optimal Hardy inequalities for Schrödinger operators on graphs*. Communications in Mathematical Physics, 358:767–790, February 2018.

### 7. Irfan Glogić

Title: *On spectral stability of self-similar solutions to nonlinear wave equations*

Abstract: The spectral approach to the stability analysis of self-similar solutions to nonlinear wave equations typically leads to non-self-adjoint problems, in contrast to the case of static solutions where the spectral analysis is entirely self-adjoint. In this talk, we consider the focusing cubic wave equation and show that, for certain special self-similar solutions, the corresponding non-self-adjoint spectral problem can nevertheless be completely resolved. By contrast, the general case remains largely open and appears to require fundamentally new spectral techniques.

### 8. Denis Grebenkov

Title: *Mixed Steklov-Neumann-Dirichlet spectral problems: asymptotic analysis and applications*

Abstract: In this talk, I consider the mixed spectral problem for the Laplace operator in a bounded domain, in which the boundary is split into three disjoint subsets, with Steklov, Neumann and Dirichlet boundary conditions. This extension of the conventional Steklov problem is known to have a positive discrete spectrum. In joint works with M. J. Ward, we investigate the asymptotic behavior of the eigenvalues and eigenfunctions of this problem in the limit when the Steklov and Dirichlet subsets shrink. Using the matched asymptotic techniques, we obtain two-term asymptotic expansions in two dimensions and three-term asymptotic expansions in three dimensions. These results reveal how the geometric structure of the domain (in particular, shapes and sizes of these subsets and their spatial configuration) affects the spectral properties. Some applications to the theory of diffusion-controlled reactions will be presented. An extension of this problem to a non-self-adjoint setting and some related open questions will be discussed.

References:

- [1] D. S. Grebenkov, Steklov-Neumann spectral problem: asymptotic analysis and applications to diffusion-controlled reactions, SIAM Multi. Model. Simul. 23, 1607-1664 (2025).
- [2] D. S. Grebenkov and M. J. Ward, Competition of small targets in planar domains: from Dirichlet to Robin and Steklov boundary condition, Eur. J. Appl. Math. 5, 1-48 (2026).
- [3] D. S. Grebenkov and M. J. Ward, The Asymptotic Analysis of Some PDE and Steklov Eigenvalue Problems with Partially Reactive Patches in 3-D (preprint in ArXiv:2509.17394).

### 9. Martin Grothaus

Title: *Weak hypocoercivity for operator semigroups with non-selfadjoint generator*

Abstract: Motivated by problems from Industrial Mathematics we further developed the concepts of hypocoercivity. The original concepts needed Poincaré inequalities and were applied to equations in linear finite dimensional spaces. Meanwhile we can treat equations in manifolds or even infinite dimensional spaces. The condition giving micro- and macroscopic coercivity we could relax from Poincaré to weak Poincaré inequalities. In this talk an overview and many examples are given.

### 10. Martin Halla

Title: *Analysis of circular waveguides*

Abstract: To simulate wave propagation in long electromagnetic waveguides (such as optical fiber amplifiers) a knowledge of the stability constant in terms of the length of the waveguide is important. The corresponding analysis relies on the properties of the modes, and in particular, if they form a Riesz basis. In this case a linear growth with respect to the length as been established in [1]. However, for bent fibers energy leaves the system and such effects are modeled with impedance-type boundary conditions. The latter make the modal eigenvalue problem non-selfadjoint and the analysis becomes more complicated. We present recent results on bent slab waveguides, showing that the growth with respect to the length stays linear [2]. Our analysis heavily relies on the concept of local form-subordination [3] and we present slight improvements to [3] concerning decay rates and the extension of results to the energy space.

## References:

- [1] M. Melenk, L. Demkowicz, and S. Henneking, "Stability analysis for electromagnetic waveguides. Part 1: Acoustic and homogeneous electromagnetic waveguides", *SIAM Journal on Mathematical Analysis*, vol. 57, no. 3, 2025, <https://doi.org/10.1137/23M1599239>.
- [2] L. Demkowicz, M. Halla, and M. Melenk, "Analysis of circular waveguides", *Oden Institute, Tech. Rep. 2*, 2025.
- [3] B. Mityagin and P. Siegl. Local form-subordination condition and Riesz basisness of root systems. *J. Anal. Math.*, 139(1):83–119, 2019.

### 11. Maryna Kachanovska

Title: *Wave Propagation in a Waveguide Filled with a Cold Strongly Magnetized Plasma*

Abstract: We consider the 2D Maxwell equations describing wave propagation in a cold strongly magnetized plasma, posed in a locally perturbed waveguide. Our goal is to analyze the spectrum of the underlying self-adjoint operator, which reduces to studying the behaviour of a boundary-value problem for a PDE with frequency-dependent coefficients. The main difficulty is that, for a range of real frequencies, the principal symbol of the partial differential operator becomes hyperbolic. A similar problem occurs in fluid mechanics ('internal gravity waves'). We show that the limiting absorption principle holds for a range  $I$  of frequencies, which depends on the geometric perturbation, by fairly elementary means, namely using the well-posedness of the Cauchy problem for the wave-type equations in moving domains and the positivity of the Dirichlet-to-Neumann maps. Next, we discuss sufficient conditions on the geometric perturbations that ensure the absence of eigenvalues outside of  $I$ . This is joint work with Dylan Machado.

### 12. Perry Kleinhenz

Title: *Geometry of wave damping on the torus*

Abstract: Energy decay rates of damped waves on the torus depend on the behavior of the damping near the undamped region and the geometry of the damped set. We will begin with an overview of these past results, which are proved via resolvent estimates of a non-self adjoint operator, and then discuss our refinement to these results. We show improved energy decay rates depending on growth of the damping near its zero set and the shape of the damped set near undamped geodesics. The proof invokes an averaging result to reduce the problem to estimating the damping averaged along certain rational directions. Time permitting we will show that damping sets which attain these improvements are generic among polygons and smooth curves.

### 13. Yehuda Pinchover

Title: *Strictly positive optimal Hardy weight via Green potential*

Abstract: We construct strictly positive optimal Hardy-type inequalities for  $p$ -Schrödinger operators with a potential term. As applications, we obtain for the linear case, some spectral results, including the completeness of the Agmon metric and Rellich-type inequalities.

### 14. Nico Michele Schiavone

Title: *Uniform resolvent estimates for the Heisenberg sublaplacian*

Abstract: We present global bounds for solutions to both stationary and time-dependent Schrödinger equations associated with the sublaplacian in the the Heisenberg group, as well as its fractional powers and conformally invariant fractional powers. The key ingredient in our approach is a new abstract uniform weighted resolvent estimate, derived using the method of weakly conjugate operators—a variant of Mourre's commutator method—together with Hardy-type inequalities on the Heisenberg group.

Resolvent estimates and the Helmholtz equation represent some of the strongest connections between Fourier Analysis, Partial Differential Equations, and Spectral Theory. In fact, as an application of our uniform resolvent estimates, we derive Kato-type smoothing effects for the time-dependent Schrödinger equation, and the spectral stability of the sublaplacian perturbed by complex decaying potentials that satisfy a subordination condition.

Joint work with Luca Fanelli, Haruya Mizutani, and Luz Roncal.

### 15. František Štampach

Title: *The spectral pair for Schrödinger operators with complex integrable potentials*

Abstract: I will answer a question which I was asked two years ago at this conference by providing a more detailed spectral and scattering-type description of the spectral pair of Schrödinger operators on the half-line with a complex integrable potential. The talk is based on a collaboration with A. Pushnitski.

### 16. Christiane Tretter

Title: *Essential numerical ranges*

Abstract: In this talk various essential numerical range concepts will be presented and compared. Their potential to capture spectral pollution and spectral invisibility makes them a valuable tool, in particular, for applications to partial differential operators and, more specifically, to non-selfadjoint Schrödinger operators.

(joint work with Sabine Bögli, Durham, and Marco Marletta, Cardiff)

### 17. Nicolas Weber

Title: *Operator families in a weak coupling setting*

Abstract: We study the existence, absence, multiplicity and uniqueness of eigenvalues emerging from the threshold of the essential spectrum of the Schrödinger operator

$$-\Delta - \beta V \quad \text{in } L^2(\mathbb{R}^2)$$

as the complex coupling parameter  $\beta$  tends to zero, under suitable integrability assumptions on the complex potential  $V$ . Our results can be viewed as natural non-self-adjoint extensions of the well-known classical weak coupling phenomenon for

self-adjoint Schrödinger operators with real-valued potentials going back half a century to the seminal work of Simon [1]. We further extend this analysis to more general operator families.

This talk is based on joint work with J. Behrndt, M. Holzmann and P. Siegl.

References:

[1] B. Simon, *The Bound State of Weakly Coupled Schrödinger Operators in One and Two Dimensions*, Ann. Physics, **97** (1976), 279-288.

### 1. Calina Ana Maria Adelina

Title: *Improved  $H^2$  Kato regularity for the Laplacian with relatively bounded perturbations*

Abstract: We present regularity results for solutions of elliptic equations with singular coefficients of type  $-\Delta v + \lambda \frac{x \cdot \nabla v}{|x|^2} = f$  and  $-\Delta v + \lambda \frac{v}{|x|^2} = f$ . In these cases we refine Kato's classical regularity result for the Laplacian under relatively bounded perturbations, where Hardy–Rellich-type inequalities play a crucial role. The regularity strongly depends on the value of the spectral parameter  $\lambda$ .

This talk is based on joint work with Cristian Cazacu. Partially supported by the doctoral fellowship of the University of Bucharest and from the NSF–UEFISCDI grant ROSUA-2024-0001 at IMAR.

### 2. Martin Averseng

Title: *Quantum tunneling through complex-valued potentials*

Abstract: Since the early developments of quantum physics, it is well-known that electrons can “tunnel” through potential barriers : that is, electrons can be found in regions that are inaccessible to a corresponding classical particle with the same energy. A rigorous mathematical description of this phenomenon was given in the 1980s by Harell, Simon, Helffer and Sjöstrand, in the case of a symmetric double-well potential in the semiclassical regime in arbitrary dimension. They showed that electrons initially localized in one of the wells will be found with high probability in the opposite well after a time proportional to the inverse of the eigenvalue-gap of the governing Schrödinger operator (i.e., the gap between its two smallest eigenvalues). They obtained precise estimates of this gap in terms of the so-called “Agmon distance” between the wells. In physical applications, it is also common to consider complex-valued potentials (for instance to model dissipation effects) but in this case, a mathematical description of tunneling like above is not currently known. The goal of this talk is to tackle this problem in dimension 1. The main result is a precise eigenvalue gap estimate that extends the known one for real-valued potentials.

This is joint work with Frédéric Hérau, Nicolas Frantz and Nicolas Raymond.

### 3. Jacob Barnett

Title: *Exceptional Flow*

Abstract: Given a parametric Hamiltonian, how can we systematically locate its exceptional points? We show that this problem can be reformulated as a nonlinear least-squares minimization problem and solved using iterative methods. The resulting flows may be tailored to converge to exceptional points of arbitrary order (when they exist). Additionally, such a flow generates a complete Jordan chain of generalized eigenvectors and the corresponding defective eigenvalue. Computations may be efficiently performed with fewer elementary operations when the Hamiltonian is sparse. We highlight the efficiency of this approach by comparing it with traditional methods based on manipulating characteristic polynomials; for example, unlike traditional methods, directly find exceptional eigenvalues without needing to compute the rest of the Hamiltonian's spectrum.

### 4. Ujjal Das

Title: *Behavior of the ground state energy under weak perturbation of critical quasilinear operators on  $\mathbb{R}^N$*

Abstract: We will consider a critical quasilinear operator  $-\Delta_p[u] + V|u|^{p-2}u$  in  $\mathbb{R}^N$  perturbed by a weakly coupled potential. For  $N > p$  we find the leading asymptotic of the lowest eigenvalue of such an operator in the weak coupling limit separately for  $N > p^2$  and  $N \leq p^2$ . Based on my recent work with Hynek Kovarik and Yehuda Pinchover.

### 5. Catherine Drysdale

Title: *Computation and Verification of Spectra for Non-Hermitian Systems*

Abstract: We establish a deep connection between quantum mechanics and computation, revealing fundamental limitations for algorithms computing spectra, especially in non-Hermitian settings. Introducing the concept of locally trivial pseudospectra (LTP), we show such assumptions are necessary for spectral computation. LTP adapts dynamically to system energies, enabling spectral analysis across a broad class of challenging non-Hermitian problems. Exploiting this framework, we overcome a longstanding obstacle by computing the eigenvalues and eigenfunctions of the imaginary cubic oscillator  $H_B = p^2 + ix^3$  with rigorous error bounds and no spurious modes—yielding, to our knowledge, the first such error-controlled result. We confirm, for instance, the 100th eigenvalue as 627.6947122484365113526737029011536... Here, truncation-induced PT-symmetry breaking causes spurious eigenvalues—a pitfall our method avoids, highlighting the link between truncation and physics. Finally, we illustrate the approach's generality via spectral computations for a range of physically relevant operators. This letter provides a rigorous framework linking computational theory to quantum mechanics and offers a precise tool for spectral calculations with error bounds.

Joint work with Matthew Colbrook and Michael T. M. Woodley.

### 6. Patrick Ebnicher

Title: *Neumann Realisation of Schrödinger Operators with Unbounded Complex Potentials*

Abstract: We study Schrödinger operators with complex-valued, possibly unbounded potentials  $V$  on unbounded domains subject to Neumann boundary conditions. Assuming that  $V$  has a non-negative real part and suitable  $L^p_{\text{loc}}$  regularity up to the boundary, we prove that the associated operators are  $m$ -accretive.

To rigorously impose the boundary condition at the operator level, we construct a surjective Neumann trace adapted to unbounded domains and unbounded complex potentials.

The proof relies on a complex-valued version of Kato's inequality valid up to the boundary. Our results extend known properties of non-selfadjoint Schrödinger operators beyond classical sectorial and standard form-based approaches.

### 7. Majed Fakhfakh

Title: *Generalized weakly demicompact linear relations and their spectral properties*

Abstract: In this talk, we extend the concept of generalized weakly demicompact operators on linear relations and we display some outstanding results. Moreover, we address the theory of Fredholm and upper semi-Fredholm relations attempting to enact a connection with them.

### 8. Florian Fischer

Title: *Characterizations of  $p$ -Parabolicity on Graphs*

Abstract: We study  $p$ -energy functionals on infinite locally summable graphs for  $p \in (1, \infty)$  and discuss that many well-known characterizations for a parabolic space are also true in this discrete, non-local and non-linear setting. Among them are an Ahlfors-type, a Kelvin-Nevanlinna-Royden-type, a Khas'minskiĭ-type and a Poincaré-type characterization. Some of them we will study in detail. We also illustrate some applications and describe examples of graphs which are locally summable but not locally finite. The talk is based on joint work with Andrea Adriani (Rabat) and Alberto Setti (Como).

### 9. Nicolas Frantz

Title: *Spectral analysis of a semi-classical Bloch-Torrey operator*

Abstract: The Bloch-Torrey operator  $(-h^2\Delta + ix)$  is the non-selfadjoint differential operator that governs the time-evolution of the magnetization of spin-bearing particles in a body submitted to a magnetic field. This operator is central in the modelling of diffusion MRI (A medical technique especially used for brain imaging). In particular the localization of its eigenfunctions, which is important for applications, is not currently understood. The purpose of this talk is to understand this localization by means of an Agmon estimate. The main tool is the construction of a parametrix using the symbolic calculus of operator-valued pseudodifferential operators.

This is a joint work with Martin Averseng, Frédéric Hérau and Nicolas Raymond.

### 10. Jan Havel

Title: *Local density of states for 2D-Schrödinger operators with  $\delta$ -point interactions*

Abstract: In physics, the local density of states is a quantity that measures the number of available quantum states per unit energy at a given spatial location. In the case of two-dimensional Schrödinger operators with delta-point interactions, we provide a rigorous definition for positive scattering energies. We also derive formulas showing that the local density of states can be computed either from the Green's function via the limiting absorption principle or from generalised eigenfunctions using the eigenfunction expansion theorem.

### 11. Mikuláš Kučera

Title: *Spectrum of the wave equation with Dirac damping on a compact star graph*

Abstract: We consider the wave equation with a distributional Dirac damping and Dirichlet boundary conditions on a compact interval. It is shown that the spectrum of the corresponding wave operator is fully determined by zeroes of an entire function. Consequently, a considerable change of spectral properties is shown for certain critical values of the damping parameter. We also derive a definitive criterion for the Riesz basis property of the root vectors for an arbitrary placement of a complex-valued Dirac damping. Finally, we consider a generalisation of the problem for compact star graphs and provide insight into the essence of the critical damping constant.

### 12. Vincent Louatron

Title: *Resonances for matrix-valued Schrödinger operators via normal forms*

Abstract: Matrix-valued Schrödinger operators naturally arise when studying the molecular predissociation phenomena in quantum chemistry. More precisely, the molecular hamiltonian describing the molecule is reduced in the Born-Oppenheimer approximation to an effective, matrix-valued Schrödinger operator  $P$ . The imaginary part of the resonances of  $P$  is known to be proportional to the inverse of the half-life of the molecule. In the semiclassical limit ( $h =$  ratio of electronic masses over nucleic masses), the distribution of the resonances is governed by the underlying classical trajectories of the effective operator. When those trajectories cross, this quantity is polynomial in  $h$  and can be computed using a so-called microlocal connection formula at the crossing point of the classical trajectories. In this poster, we recover the resonances of  $P$  using a recent result (joint work with K. Higuchi, K. Taira) which uses normal forms to reduce the computation of the microlocal connection formula for  $P$  to that of a matrix-valued, first-order differential operator. This method also applies to more general matrix-valued pseudo-differential operators, including  $P$ .

### 13. Dylan Machado

Title: *Limiting absorption principle for a waveguide filled with a hyperbolic medium*

Abstract: Hyperbolic electromagnetic metamaterials are artificial media which exhibit a lot of interesting physics phenomena such as negative refraction or rainbow-trapping [3], [4]. The simplest mathematical model stems from electromagnetic wave propagation in a strongly magnetised cold plasma described by 2D Maxwell equations which, in their time harmonic form, read

$$(1) \quad \varepsilon(\omega)^{-1} \partial_x^2 u + \partial_y^2 u + \omega^2 u = 0$$

where  $\varepsilon(\omega)$  is the dielectric permittivity and  $\omega \in \mathbb{R}$  denotes the frequency. In hyperbolic media,  $\varepsilon(\omega)$  may be negative for a range of frequencies, thus the above equation becomes hyperbolic [1], hence the name. A similar type of problem occurs as well in fluid mechanics [2].

We consider the equation (1) posed in a straight waveguide with a locally varying boundary. The goal of this work in progress is to understand the spectrum of the first-order operator leading to the non-linear eigenvalue problem (1).

We first study the case of a straight boundary with techniques that may be adapted to the case of a varying boundary. It can be shown that the problem becomes elliptic for  $\omega \in \mathbb{C} \setminus \mathbb{R}$ . We exploit this fact to prove a certain control on the traces of the solution with absorption. Next, we switch the viewpoint and treat the equation (1) as a Cauchy problem for the wave equation with well-controlled initial data (traces of the solution) in a moving domain. This allows us to use techniques adapted to evolution problems and prove the limiting absorption principle in the case when the underlying evolution problem is well-posed. A sufficient condition on the well-posedness is that the boundary of the moving domain moves slower than waves in the media, which now becomes a geometric condition on the frequency.

Finally, the phenomenon of propagation of singularities arises from the hyperbolic nature of (1). In the straight case, it can be established by computing an adapted representation of Green's function. Numerical simulations reveal patterns created by the propagation of singularities.

This is a joint work with Maryna Kachanovska.

References:

- [1] P. Ciarlet and M. Kachanovska. A mathematical study of a hyperbolic metamaterial in free space. *SIAM J. Math. Anal.*, 54:2216–2250, 2022.
- [2] S. Dyatlov, J. Wang, and M. Zworski. Mathematics of internal waves in a 2d aquarium. *Analysis & PDE*, 18:1–92, 2025.
- [3] T. Krauss. Why do we need slow light ? *Nature photonics*, 2008.
- [4] A. Poddubny, I. Iorsh, P. Belov, et al. Hyperbolic metamaterials. *Nature Photon*, 7:948–957, 2013.

#### 14. Frederick Moscatelli

Title: *Mode stability of self-similar wave maps outside of corotational symmetry*

Abstract: The wave maps equation is a nonlinear geometric wave equation, which generalizes the free wave equation to curved targets. In the case of spherical targets there exists an explicit self-similar solution that exhibits finite-time blowup. This solution is corotational and within the class of corotational functions its stability has been established in the last 15 years. The stability outside of corotational symmetry was proved recently. We present the first step of this proof, which constitutes the main technical difficulty, namely the establishment of mode stability. After applying tools from Lie algebra theory, this boils down to the analysis of spectral ODEs. Due to the nonself-adjoint nature of the problem, this is not a standard Sturm-Liouville problem and hence requires a different approach.

This is joint work with Roland Donninger.

#### 15. Rossano Sannipoli

Title: *Geometrical bounds for the Torsion with Robin boundary conditions*

Abstract: We study shape functionals involving the torsional rigidity with Robin boundary conditions in the class of convex sets. We derive an upper bound in terms of norms of the distance function from the boundary, obtaining a Robin-type Makai inequality. We also establish quantitative estimates for related functionals and show that slab-like domains arise as optimal asymptotic shapes.

#### 16. Peter Schlosser

Title: *The Neumann trace for unbounded Lipschitz domains*

Abstract: For a bounded smooth domains  $\Omega$ , the Neumann trace operator is classically defined as the bounded extension  $\tau_N : H^2(\Omega) \rightarrow H^{\frac{1}{2}}(\partial\Omega)$  of the normal derivative acting on smooth functions.

It is then a well known result that this Neumann trace can be extended to an operator  $\tau_N : H_{\Delta}^1(\Omega) \rightarrow H^{-\frac{1}{2}}(\partial\Omega)$ , via the second Green's identity

$$(\tau_N u, \tau_D v)_{H^{-\frac{1}{2}}(\partial\Omega) \times H^{\frac{1}{2}}(\partial\Omega)} = (\nabla u, \nabla v)_{L^2(\Omega)} - (-\Delta u, v)_{L^2(\Omega)}.$$

Here  $H_{\Delta}^1(\Omega)$  is the space of all function  $f \in L^2(\Omega)$ , such that also the distributional derivative  $\Delta f \in L^2(\Omega)$  is square integrable.

The aim of the present work is to further extend this notion of Neumann trace to:

- Unbounded Lipschitz surfaces;
- Functions  $f \in H^{\frac{1}{2}+\varepsilon}(\Omega)$  with  $\Delta f$  only in some dense subset of  $L^2(\Omega)$ ;
- Functions  $f \in H^{\frac{1}{2}+\varepsilon}(\Omega)$ , with  $-\Delta + V$  in some dense subset of  $L^2(\Omega)$ . Here, the potential  $V$  is unbounded, contained in some space  $L^p(\Omega)$ .

#### 17. Sandeep Kumar Soni

Title: *Extension theory for symmetric positive systems of first order PDEs*

Abstract: With the aim to give a suitable setting for studying mixed-type equations (e.g. the Tricomi equation) K. O. Friedrichs introduced positive symmetric systems of first order PDEs in 1958, which nowadays are better known as Friedrichs systems. These systems proved to be unified approach for studying various types of PDEs.

This poster contains a brief introduction of Friedrichs systems, boundary triples and extension theory, along with the application on Maxwell's equations.

## 18. Eduard Stefanescu

Title: *Random Schrödinger operators on compact manifolds*

Abstract: We consider Schrödinger operators  $-\Delta_g + V_\omega$  on compact manifolds  $(M, g)$  with random complex potentials  $V_\omega$ , and prove Keller-type estimates for their eigenvalues. Compared to the deterministic setting considered in [J.-C. Cuenin, Eigenvalue bounds for Schrödinger operators with complex potentials on compact manifolds, Forum Math. (2025)], the bounds obtained here allow for a more precise localization of eigenvalues which are at most of the order of the inverse square of the randomization length scale. Our bounds are natural analogues of those in [J.-C. Cuenin and K. Merz, Random Schrödinger operators with complex decaying potentials, Anal. PDE 18 (2025), no. 2, 279–306] obtained in the euclidean case.

This is a joint work with Jean-Claude Cuenin and Konstantin Merz.

## 19. Matěj Tušek

Title: *Continuum limit for discrete block operator matrices*

Abstract: Building upon a recent result by H. Cornean, H. Garde, and A. Jensen concerning continuum limits of discrete Dirac operators, we extend our analysis to a wide class of block operator matrices. This class includes the bilayer graphene Hamiltonian, among others. Our main goal is to find norm estimates for the difference between the resolvents of continuous operators and their discrete counterparts embedded in the continuum in a specific way. Consequently, we investigate whether certain discretized operators converge to the original continuous operators as the mesh parameter tends to zero. While some discretization schemes lead immediately to the convergence in the generalized norm resolvent sense, others may require the addition of a suitable correction term to ensure the convergence.

## 20. Éric Vachelet

Title: *The superadiabatic projectors method applied to the spectral theory of magnetic operators*

Abstract: This talk deals with a generalization of the superadiabatic projectors method. In a general framework, the well-known superadiabatic projectors are constructed and accurately described in the case of rank one, when a remarkable factorization occurs. We apply these ideas to spectral theory and we explain how our abstract results allow to recover or improve recent results about the semiclassical magnetic Laplacian where all the subprincipal terms are not self-adjoint.

Joint work with Lino Benedetto, Clotilde Fermanian Kammerer and Nicolas Raymond.

## 21. Jakub Wacławek

Title: *Optimal discrete Hardy–Rellich–Birman inequalities*

Abstract: We study Hardy–Rellich–Birman type inequalities for an integer power  $\ell$  of the discrete Laplacian on the half-line. Starting from a parameter sequence satisfying suitable sufficient conditions, we construct an optimal weight for these inequalities in a new unified way.

For suitable choices of the parameter sequence, this method produces infinitely many explicit optimal weights. In particular, for  $\ell = 1$  we recover the optimal Hardy weight of Keller–Pinchover–Pogorzelski, while for  $\ell = 2$  we obtain new optimal discrete Rellich weights that improve upon the previously best known results of Gerhat–Krejčířík–Štampach and Huang–Ye. For  $\ell \geq 3$ , we prove a conjecture of Gerhat–Krejčířík–Štampach and identify the optimal discrete Birman weights, improving earlier results of Huang–Ye.

The approach extends to the general  $\ell^p$  setting for all  $p > 1$  and also yields the classical  $p$ -Birman inequality in both discrete and continuous settings.

This presentation is based on joint work with F. Štampach.