NSA at CIRM

Marseille, June 5–9, 2017

Schedule

Monday, June 5

08.50 - 09.00	*Welcome & Opening*
09.00 - 09.40	**Lyonell Boulton:** *A singular family of J-selfadjoint Schrödinger operators*
09.45 - 10.10	**Christian Engström:** *Non-selfadjoint operator functions and applications to plasmonics*
10.15 - 10.40	**Joe Viola:** *Understanding the Schrödinger evolution via the Hamilton flow for a (non-self-adjoint) degree-2 polynomial Hamiltonian*
10.45 - 11.15	*Coffee break*
11.15 - 11.55	**Luca Fanelli:** *Multipliers methods for Spectral Theory*
12.00 - 12.25	**Sabine Bögli:** *Schrödinger operator with non-zero accumulation points of complex eigenvalues*
12.30 - 14.30	*Lunch*
14.30 - 15.10	**Yehuda Pinchover:** *On Green functions of second-order elliptic operators on Riemannian Manifolds: the critical case*
15.15 - 15.40	**Marcel Hansmann:** *On the distribution of eigenvalues of compactly perturbed operators*
15.45 - 16.15	*Coffee break*
16.15 - 16.55	**Stanislas Kupin:** *Recent advances on the study of discrete spectra of non-selfadjoint operators*
17.00 - 17.25	**Jean-Claude Cuenin:** *Lieb-Thirring type bound for Dirac and fractional Schrödinger operators with complex potentials*
17.30 - 17.55	**Haruya Mizutani:** *Eigenvalue bounds for non-self-adjoint Schrödinger operators with the inverse square potential*
19.30	*Dinner*

Tuesday, June 6

09.00 - 09.40	**Eva Gallardo-Gutiérrez:** *The Invariant Subspace Problem:* a concrete *Operator Theory approach*
09.45 - 10.10	**Monika Winklmeier:** *Spectral decomposition of linear operators*
10.15 - 10.40	**Christian Wyss:** *Dichotomy of Hamiltonian operator matrices from systems theory*
10.45 - 11.15	*Coffee break*
11.15 - 11.55	**Zdeněk Strakoš:** *Krylov subspace methods and non-self-adjoint operators*
12.00 - 12.25	**Frank Rösler:** *A Bound on the Pseudospectrum for a Class of Non-Normal Schrödinger Operators*
12.30 - 14.30	*Lunch*
14.30 - 15.10	**Christiane Tretter:** *New spectral bounds for systems with strong damping*
15.15 - 15.40	**Jiří Lipovský:** *Eigenvalue asymptotics for the damped wave equation on metric graphs*
15.45 - 16.15	*Coffee break*
16.15 - 16.55	**Martin Kolb:** *Spectral properties of certain non-selfadjoint operators with non-local boundary condition*
17.00 - 17.25	**Malcolm Brown:** *Inverse problems for boundary triples with applications*
17.30 - 17.55	**Alexander Sakhnovich:** *Weyl theory for skew- and non-self-adjoint operators and applications*
19.30	*Dinner*

Wednesday, June 7

09.00 - 09.40	**Bernard Helffer:** *On a Schrödinger operator with a purely imaginary potential in the semiclassical limit*
09.45 - 10.10	**Nicolas Raymond:** *Non-accretive Schrödinger operators and exponential decay of their eigenfunctions*
10.15 - 10.40	**Sergey N. Tumanov:** *Exceptional Points and the Real Spectral Locus for Complex Airy Operator*
10.45 - 11.15	*Coffee break*
11.15 - 11.55	**Tom ter Elst:** *One-dimensional degenerate elliptic operators on L_p-spaces with complex coefficients*
12.00 - 12.25	**Oktay Veliev:** *On the spectral analysis of the Schrödinger operator with a periodic PT-symmetric potential*
12.30 - 14.30	*Lunch*
14.30 - 19.30	*Free afternoon*
19.30	*Dinner*

Thursday, June 8

09.00 - 09.40	**Cristina Câmara:** *Truncated Toeplitz operators*
09.45 - 10.10	**Axel Torshage:** *On spectral divisors for a class of non-selfadjoint operator polynomials*
10.15 - 10.40	**Gabriel Cardoso:** *A Riemann-Hilbert approach to black hole solutions*
10.45 - 11.15	*Coffee break*
11.15 - 11.55	**Patrick Joly:** *On the stability of linearized Eulers equations in compressible flows*
12.00 - 12.25	**Orif Ibrogimov:** *Essential spectrum of non-self-adjoint mixed-order systems of differential operators*
12.30 - 14.30	*Lunch*
14.30 - 15.10	**Eduard Feireisl:** *Acoustic waves in the incompressible limits of viscous fluids*
15.15 - 15.40	**Francesco Ferulli:** *Complexitons solution for KdV and the role of* $\sqrt{3}$
15.45 - 16.15	*Coffee break*
16.15 - 16.55	**Anne-Sophie Bonnet-Ben Dhia:** *New complex spectra associated to invisibility in waveguides*
17.00 - 17.25	**Ali Mostafazadeh:** *Dynamical theory of scattering, invisible configurations of the* ζe^{2iax} *potential, and common zeros of Bessel functions*
17.30 - 17.55	**Vladimir Lotoreichik:** *Non-self-adjoint waveguides with Robin boundary conditions*
19.30	*Conference Dinner*

Friday, June 9

09.00 - 09.40	**Nicolas Burq:** *Stabilisation of wave equations with rough damping*
09.45 - 10.10	**Florian Leben:** *A Limit-Point- and Limit-Circle Classification for* $\mathcal{PT}$*-symmetric operators*
10.15 - 10.40	**Carsten Trunk:** *Spectrum of* $\mathcal{PT}$ *symmetric operators*
10.45 - 11.15	*Coffee break*
11.15 - 11.55	**Ilya Goldsheid:** *Real and complex eigenvalues in the non-self-adjoint Anderson–Hatano-Nelson model*
12.00 - 12.25	**Stephan Schmitz:** *Block Diagonalization of Unbounded Operator Matrices*
12.30 - 14.30	*Lunch*
14.30 - 15.10	**Didier Felbacq:** *Photonic band control in a quantum metamaterial*
15.15 - 15.40	**Ivica Nakić & Krešimir Veselić:** *Perturbation of eigenvalues of the Klein Gordon operators*
15.45 - 16.15	*Coffee break*
16.15 - 16.40	**Ian Wood:** *Construction of the selfadjoint dilation of a maximal dissipative operator*
16.45 - 17.25	**Vadim Kostrykin:** *t.b.a.*
19.30	*Dinner*

Abstracts

Invited talks

1. **Anne-Sophie Bonnet-Ben Dhia**

   Title: *New complex spectra associated to invisibility in waveguides*

   Abstract: We consider an acoustic waveguide modeled as follows:

$$\begin{cases} \Delta u + k^2(1+V)u = 0 & \text{in} \quad \Omega = \mathbb{R}\times]0,1[\\ \dfrac{\partial u}{\partial y} = 0 & \text{on} \quad \partial\Omega \end{cases}$$

   where u denotes the complex valued pressure, k is the frequency and $V \in L^\infty(\Omega)$ is a compactly supported potential.

   It is well-known that they may exist non trivial solutions u in $L^2(\Omega)$, called *trapped modes*. Associated eigenvalues $\lambda = k^2$ are embedded in the essential spectrum $\mathbb{R}^+$. They can be computed as the real part of the complex spectrum of a non-self-adjoint eigenvalue problem, defined by using the so-called Perfectly Matched Layers (which consist in a complex dilation in the infinite direction) [1].

   We show here that it is possible, by modifying in particular the parameters of the Perfectly Matched Layers, to define new complex spectra which include, in addition to trapped modes, frequencies where the potential V is, in some sense, *invisible* to one incident wave.

   Our approach allows to extend to higher dimension the results obtained in [2] on a 1D model problem.

   [1] S. Hein and W. Koch, Acoustic resonances and trapped modes in pipes and tunnels, *Journal of Fluid Mechanics*, 605, (2008) pp.401-428.

   [2] H. Hernandez-Coronado, D. Krejcirik and P. Siegl, Perfect transmission scattering as a $\mathcal{PT}$-symmetric spectral problem, *Physics Letters A*, 375(22), (2011) pp.2149-2152.

2. **Lyonell Boulton**

   Title: *A singular family of J-selfadjoint Schrödinger operators*

   Abstract: The eigenvalue equation for the operator

$$H_{m,\gamma} = -\partial_r^2 + \left(m^2 - \frac{1}{4}\right)\frac{1}{r^2} + \frac{\gamma^2}{4}r^2$$

   can be exactly solved in terms of known special functions for any $m, \gamma \in \mathbb{C}$. This operator appears naturally in many situations. In particular, it is obtained when we separate the variables in a rotationally symmetric harmonic oscillator in two or more dimensions. It can also be seen as a case that just falls outside the Schrödinger operators describing spectral determinants of Q-operators. There exists a vast literature about this operator, dealing mainly with algebraic aspects.

   In this talk we will examine a family of closed operators realising $H_{m,\gamma}$ in the Hilbert space $L^2[0,\infty[$, depending on the two complex parameters for the maximal region $\Re(m) > -1$ and $\Re(\gamma) \geq 0$. We will investigate thoroughly the family $H_{m,\gamma}$. In particular, we will show that it is a holomorphic family of J-selfadjoint operators, find its spectrum, compute its resolvent and enclose its numerical range. We will then discuss open problems connected to phase transition properties, resolvent norm and stability of perturbations.

   This research has been conducted in collaboration with Jan Dereziński.

3. **Nicolas Burq**

   Title: *Stabilisation of wave equations with rough damping*

   Abstract: For the damped wave equation on a compact manifold with continuous dampings, the geometric control condition is necessary and sufficient for uniform stabilization. In this talk, I will present some results exhibiting examples for which we have necessary and sufficient conditions even for non smooth damping.

4. **Cristina Câmara**

   Title: *Truncated Toeplitz operators*

   Abstract: Toeplitz matrices and operators constitute one of the most important and widely studied classes of non-self-adjoint operators. In this talk we consider truncated Toeplitz operators, a natural generalisation of finite Toeplitz matrices. They appear in various contexts, such as the study of finite interval convolution equations, signal processing, control theory, diffraction problems, hydrodynamics, elasticity, and they play a fundamental role in the study of complex symmetric operators. We will focus mainly on their invertibility and Fredholmness properties, showing in particular that they are equivalent after extension to block Toeplitz operators, and how this can be used to study the spectra of several classes of truncated Toeplitz operators.

5. **Tom ter Elst**

   Title: *One-dimensional degenerate elliptic operators on L_p-spaces with complex coefficients*

   Abstract: Let $c\colon \mathbb{R} \to \mathbb{C}$ be a bounded Lipschitz continuous function which takes values in a sector. We consider the divergence form operator $A = -\frac{d}{dx}\, c\, \frac{d}{dx}$ in $L_2(\mathbb{R})$. We characterize for which $p \in [1, \infty)$ the semigroup generated by $-A$ extends consistently to a contraction C_0-semigroup on $L_p(\mathbb{R})$ and for those p we characterize when $C_c^\infty(\mathbb{R})$ is a core for the generator in $L_p(\mathbb{R})$.

   This is joint work with Tan Do.

6. **Luca Fanelli**

   Title: *Multipliers methods for Spectral Theory*

   Abstract: We will review some recent results concerning the following two topics, which are strongly related to each other: a priori estimates for the Helmholtz Equation and absence of eigenvalues for perturbations of the free Hamiltonian. We will present a multiplier method which can be nowadays seen under a quite general point of view, also involving "Morawetz-type" estimates for the evolution Schrödinger equation.

7. **Eduard Feireisl**

   Title: *Acoustic waves in the incompressible limits of viscous fluids*

   Abstract: We discuss the problems of propagation, dispersion, and annihilation of acoustic waves in the low Mach number limit for the Navier-Stokes system describing the motion of a compressible viscous fluids. We distinguish the well and ill prepared initial data and show how these influence the acoustic waves propagation. Several extensions to more complex systems are also considered.

8. **Didier Felbacq**

   Title: *Photonic band control in a quantum metamaterial*

   Abstract: We present a metamaterial made of a periodic collection of dielectric resonators in which a quantum oscillator (denoted QO in the following) is inserted. The geometry at stake here is much more complicated than the textbook 1D cavity usually dealt with theoretically in quantum optics. We provide a treatment essentially semi-classical, based on the scattering matrix non-perturbative approach, in order to investigate the various effects that could be expected to exist in such structures.The theoretical methods used are the Feshbach projection method associated with multiple scattering theory. First, the phenomenology for one scatterer with a QO inserted is presented, then the collective behavior of a finite periodic set of such scatterers is investigated and it is shown that it is possible to open and close a conduction band according to the state of the oscillators when the inserted quantum oscillators are put in the inversion regime by means of a pump field. They add gain to the system, allowing to reach the amplification regime in the vicinity of the Mie resonances of the dielectric resonators. When the transition frequency is situated at the photonic band gap edge, it creates switchable conducting modes within the bandgap. From a methodological point of view, we investigate the relevance of the use of permittivity whose imaginary part is negative to describe gain. We show that this leads to a field that grows exponentially in time. This results demonstrates the limited relevance of this model beyond the early times of the processes at stake. Possible remedies to this problem are then described.

9. **Eva Gallardo-Gutiérrez**

   Title: *The Invariant Subspace Problem: a* concrete *Operator Theory approach*

   Abstract: The Invariant Subspace Problem for (separable) Hilbert spaces is a long-standing open question that traces back to Jonhn Von Neumann's works in the fifties asking, in particular, if every bounded linear operator acting on an infinite dimensional separable Hilbert space has a non-trivial closed invariant subspace. Whereas there are well-known classes of bounded linear operators on Hilbert spaces that are known to have non-trivial, closed invariant subspaces (normal operators, compact operators, polynomially compact operators,...), the question of characterizing the lattice of the invariant subspaces of just a particular bounded linear operator is known to be extremely difficult and indeed, it may solve the Invariant Subspace Problem.

   In this talk, we will focus on those *concrete operators* that may solve the Invariant Subspace Problem, presenting some of their main properties, exhibiting old and new examples and recent results about them obtained in collaboration with Prof. Carl Cowen (Indiana University-Purdue University).

10. **Ilya Goldsheid**

    Title: *Real and complex eigenvalues in the non-self-adjoint Anderson–Hatano-Nelson model*

    Abstract: In 1997 N. Hatano and D. Nelson introduced a non-self-adjoint version of the Anderson model and discovered, numerically, several remarkable properties of its spectrum (Phys. Rev. B56, 8651–8673). The remarkable behaviour of complex eigenvalues of this model was explained in a series of three papers by B. Khoruzhenko and myself (see Phys. Rev. Lett. 80, 2897-2900 (1998); Electron. J. Probability 5, 26p, (2000); Commun. Math. Phys. 238, 505 - 524 (2003)). It is now relatively well understood. However, the reasons for the behaviour of the real eigenvalues of this mode remained somewhat mysterious. In my talk, I will briefly review the previously obtained results and discuss very recent progress, in joint work with A. Sodin, concerned with the description of the behaviour of the real eigenvalues.

11. **Bernard Helffer**

    Title: *On a Schrödinger operator with a purely imaginary potential in the semiclassical limit*

    Abstract: We consider the operator $\mathcal{A}_h = -h^2\Delta + iV$ in the semi-classical limit $h \to 0$, where V is a smooth real potential with no critical points. We obtain both the left margin of the spectrum, as well as resolvent estimates on the left side of this margin. We extend here previous results obtained for the Dirichlet realization of $\mathcal{A}_h$ by removing significant limitations that were formerly imposed on V. In addition, we apply our techniques to the more general Robin boundary condition and to a transmission problem which is of significant interest in physical applications.

12. **Patrick Joly**

    Title: *On the stability of linearized Eulers equations in compressible flows*

    Abstract: We address in this work the question of the stability analysis of compressible linearized Eulers equations in a duct filled with a fluid in laminated stationary flow. The question of interest concerns the influence of the flow profile on the stability of the linearized evolution problem. Most existing results concern the case of incompressible fluids. In the compressible case, which is the relevant case as far as acoustic wave propagation is concerned, we revisit this problem for thin ducts by means of a small thickness asymptotic analysis. One is then reduced to analyze a family of non-local and non self-adjoint 1D problems. Several stability and instability results will be presented with numerical illustrations and open questions will be mentioned. This presentation covers joint works with Anne-Sophie Bonnet-Ben Dhia, Marc Durufle, Lauris Joubert and Ricardo Weder.

13. **Martin Kolb**

    Title: *Spectral properties of certain non-selfadjoint operators with non-local boundary condition*

    Abstract: We focus on a class of non-selfadjoint operators, which appear as generators of diffusions in a bounded domain with random jumps from the boundary. In the talk I discuss different interesting spectral theoretic aspects, which have been derived so far using probabilistic as well as analytic techniques.

14. **Vadim Kostrykin**

    Title: *t.b.a.*

    Abstract: t.b.a.

15. **Stanislas Kupin**

    Title: *Recent advances on the study of discrete spectra of non-selfadjoint operators*

    Abstract: In this talk, we present general methods of the study of the discrete spectrum of a non-selfadjoint operator. An extensive list of recent developments as well as applications to operators of mathematical physics (i.e., Jacobi matrices, Schrdinger-type operators (including differential operators on certain waveguides) etc.) will be discussed.

    A part of this talk is based on results obtained jointly with Ph. Briet, V. Bruneau and L. Golinskii.

16. **Yehuda Pinchover**

    Title: *On Green functions of second-order elliptic operators on Riemannian Manifolds: the critical case*

    Abstract: Let P be a second-order, linear, elliptic operator with real coefficients which is defined on a noncompact and connected Riemannian manifold M. It is well known that the equation $Pu = 0$ in M admits a positive supersolution which is not a solution if and only if P admits a unique positive minimal Green function on M, and in this case, P is said to be subcritical in M. If P does not admit a positive Green function but admits a global positive solution, then such a solution is called a (Agmon) ground state of P in M, and P is said to be critical in M.

    We prove for a critical operator P in M, the existence of a Green function which is dominated above by the ground state of P away from the singularity. Moreover, in a certain class of Green functions, such a Green function is unique, up to an addition of a product of the ground states of P and P^*. This result extends and sharpens the celebrated result of Peter Li and Luen-Fai Tam concerning the existence of a symmetric Green function for the Laplace-Beltrami operator on a smooth and complete Riemannian manifold M.

    This is a joint work with Debdip Ganguly.

17. **Zdeněk Strakoš**

    Title: *Krylov subspace methods and non-self-adjoint operators*

    Abstract: This presentation will consider Krylov subspace methods for numerical solution of equations $\mathcal{G}x = b$ on a Hilbert space, where $\mathcal{G}$ is a linear invertible operator. Unless mentioned otherwise, we will consider mathematical properties of the methods assuming exact computations.

    Krylov subspace methods are inherently related to the method of moments. At the step n they implicitly construct a finite dimensional approximation $\mathcal{G}_n$ of $\mathcal{G}$ with the desired approximate solution x_n defined by (for simplicity of notation we consider the initial guess $x_0 = 0$)

    $$x_n := p_{n-1}(\mathcal{G}_n)\, b \;\approx\; x = \mathcal{G}^{-1} b\,,$$

    where $p_{n-1}(\lambda)$ is a particular polynomial of degree at most $n-1$, and $\mathcal{G}_n$ is obtained by restricting and projecting $\mathcal{G}$ onto the nth Krylov subspace

    $$K_n(\mathcal{G}, b) := \operatorname{span}\{b, \mathcal{G}b, \ldots, \mathcal{G}^{n-1}b\}.$$

    We have to deal with two basic mathematical questions (and, in addition to that, with other theoretical and computational issues related to various numerical techniques).

    (a) How fast x_n, $n = 1, 2, \ldots$ approximate the desired solution x ?

    (b) Which characteristics of $\mathcal{G}$ and b can be used in investigating and resolving the first question?

The way these questions are approached depends to a large extent on the departure of the operator $\mathcal{G}$ from normality. In particular, for $\mathcal{G}$ self-adjoint the answer to the second question is based on the spectral information, and connections to orthogonal polynomials, Jacobi matrices, continued fractions, classical moment problems, and Gauss-Christoffel quadrature are convenient in addressing the first question. For $\mathcal{G}$ non-self-adjoint, and, in particular, for $\mathcal{G}$ far from normal, the second question remains widely open and the first question is far from being satisfactorily understood. In practice we can not afford many iteration steps, which severely limits usefulness of asymptotic analysis. This makes the problem hard. The same is true also in the finite dimensional (matrix) case.

The talk will recall several results in infinite dimensional as well as in finite dimensional setting.

18. **Christiane Tretter**

Title: *New spectral bounds for systems with strong damping*

Abstract: In this talk new enclosures for the spectra of operators associated with second order Cauchy problems are presented for non-selfadjoint damping. Our new results yield much better bounds than the numerical range of these non-selfadjoint operators for both uniformly accretive and sectorial damping.

(joint work with B. Jacob, Carsten Trunk and H. Vogt)

Contributed talks

1. **Sabine Bögli**

Title: *Schrödinger operator with non-zero accumulation points of complex eigenvalues*

Abstract: In the 1960s Pavlov studied Schrödinger operators on the half-line with potentials that decay at infinity, subject to Robin boundary conditions at the endpoint. Using inverse spectral theory, he proved the existence of a real potential and a non-selfadjoint boundary condition so that the Schrödinger operator has infinitely many non-real eigenvalues that accumulate at an arbitrary prescribed point of the essential spectrum (the positive half-line). Since then, it has been an open question whether these results can be modified so that the non-selfadjointness is not coming from the boundary conditions but from a non-real potential. In this talk we consider Schrödinger operators on the whole Euclidean space (of arbitrary dimension) or on the half-space, subject to real Robin boundary conditions. I will present the construction of a non-real potential that decays at infinity so that the corresponding Schrödinger operator has infinitely many non-real eigenvalues accumulating at every point of the essential spectrum.

2. **Malcolm Brown**

Title: *Inverse problems for boundary triples with applications*

Abstract: We discusses the inverse problem of how much information on an operator can be determined, or detected from measurements on the boundary. Our focus is on non-selfadjoint operators and their detectable subspaces (these determine the part of the operator visible from boundary measurements).

We show results in an abstract setting, where we consider the relation between the M-function (the abstract Dirichlet to Neumann map or the transfer matrix in system theory) and the resolvent bordered by projections onto the detectable subspaces. More specifically, we investigate questions of unique determination, reconstruction, analytic continuation and jumps across the essential spectrum.

The abstract results are illustrated by examples of Schrödinger operators, matrix-differential operators and, mostly, by multiplication operators perturbed by integral operators (the Friedrichs model), where we use a result of Widom to show that the detectable subspace can be characterized in terms of an eigenspace of a Hankel-like operator.

3. **Gabriel Cardoso**

Title: *A Riemann-Hilbert approach to black hole solutions*

Abstract: We consider the dimensional reduction of gravitational theories in four dimensions down to two dimensions. The space of solutions of the resulting equations of motion (non-linear PDEs in two dimensions) may be studied by first recasting these equations in terms of an auxiliary linear system, called the Breitenlohner-Maison linear system, and subsequently studying the latter by solving an

associated Riemann-Hilbert matrix factorization problem. We use this approach to obtain classes of black hole solutions.

Work in collaboration with Cristina Câmara, Thomas Mohaupt and Suresh Nampuri.

4. **Jean-Claude Cuenin**

Title: *Lieb-Thirring type bound for Dirac and fractional Schrödinger operators with complex potentials*

Abstract: We present Lieb-Thirring type bounds for fractional Schrödinger operators and Dirac operators with complex-valued potentials [1]. The spectral bounds hold for discrete as well as for embedded eigenvalues, and we provide counterexamples for the latter showing that our results are essentially sharp. The main technical tools are uniform resolvent estimates in Schatten-Von Neumann classes for the unperturbed operator. These are proved for a large class of kinetic energy operators under a natural curvature assumption on the characteristic set of the corresponding symbol. Estimates of this type have a long tradition in harmonic analysis, and their usefulness for non-selfadjoint spectral problems has first been recognized by R. L. Frank [3]. Going beyond the translation-invariant case, we also consider cases where the unperturbed operator is allowed to have an unbounded electromagnetic background field [2].

[1] J.-C. Cuenin. Eigenvalue bounds for Dirac and fractional Schrödinger operators with complex potentials. *Journal of Functional Analysis*, 2016 (in print).

[2] J.-C. Cuenin and C. E. Kenig. L^p resolvent estimates for magnetic Schrödinger operators with unbounded background fields. *Communications in Partial Differential Equations*, 2016 (in print).

[3] Rupert L. Frank. Eigenvalue bounds for Schrödinger operators with complex potentials. *Bull. Lond. Math. Soc.*, 43(4):745–750, 2011.

5. **Christian Engström**

Title: *Non-selfadjoint operator functions and applications to plasmonics*

Abstract: Operator functions whose values are a Maxwell operator are often used to describe applications in optics including metal-dielectric resonators and metamaterials. In the non-magnetic case, piecewise constant material properties are characterized by the material model

$$\epsilon(\cdot,\omega) = \sum_{\ell=1}^{L} \epsilon_\ell(\omega)\chi_{\Omega_\ell}(\cdot) \quad \omega \in \mathbb{C} \setminus P,$$

where P is the set of poles of the complex functions ϵ_ℓ, $\ell = 1, \ldots, L$, and $\Omega = \Omega_1 \cup \cdots \cup \Omega_L$ is a bounded domain. The values of the Maxwell operator-valued function are then

$$\mathcal{S}(\omega) = A - \omega^2 \sum_{\ell=1}^{L} \epsilon_\ell(\omega)\chi_{\Omega_\ell}, \quad \operatorname{dom} \mathcal{S}(\omega) = \operatorname{dom} A, \quad \omega \in \mathbb{C} \setminus P,$$

where $A := \operatorname{curl}\operatorname{curl}$ on its natural domain is self-adjoint. In this talk, we consider spectral properties of $\mathcal{S}(\cdot)$. In particular, we study the case when ϵ_ℓ are complex rational functions of Drude-Lorentz type.

The talk is based on a joint work with Axel Torshage.

6. **Francesco Ferulli**

Title: *Complexitons solution for KdV and the role of* $\sqrt{3}$

We investigate the time behavior of a class of traveling waves for the complex KdV equation. This equation is naturally linked to a one dimensional non-selfadjoint Schrödinger operator by means of its integrability. We observe that the time behavior of these new solutions depends heavily on the location of the spectral parameter in three regions of the complex plane individuated by the line $\Re z = \sqrt{3}\Im z$. We are also interested in the interaction of two such sort of solutions and on long time asymptotic behavior.

7. **Marcel Hansmann**

   Title: *On the distribution of eigenvalues of compactly perturbed operators*

   Abstract: The distribution of eigenvalues of compact operators is a classical and well-studied subject. For instance, in 1949 Weyl has shown that if the sequence of singular numbers of a compact Hilbert space operator is in $l_p(\mathbb{N})$, then the same is true of its sequence of eigenvalues. To mention another example, Grothendieck showed in 1955 that the sequence of eigenvalues of a nuclear operator on a general Banach space is always in $l_2(\mathbb{N})$. Many more examples could be added to this list and it is fair to say that the eigenvalue distribution of compact operators is by now very well understood.

   However, the same cannot be said for *compactly perturbed* operators. That is, if A is some known bounded operator and K is compact, how fast do the discrete eigenvalues of $A + K$ accumulate at the (essential) spectrum of A? In this talk, we will present some old and new results for this generalized problem, focusing on the case of operators on general Banach spaces.

   This talk is partly based on joint work with M. Demuth, F. Hanauska (Clausthal) and G. Katriel (Karmiel).

8. **Orif Ibrogimov**

   Title: *Essential spectrum of non-self-adjoint mixed-order systems of differential operators*

   Abstract: We shall present some recent work on the essential spectrum of *non-self-adjoint mixed-order* systems of (partial) differential operators. If the corresponding operator matrix contains a zero-order diagonal entry, then the essential spectrum has a local origin that can be traced to points where the Douglis-Nirenberg ellipticity breaks down. Moreover, it is well-known that singularity at infinity or boundary of underlying domain may cause an additional branch of the essential spectrum. While, the latter phenomenon is more or less well understood for a large class of mixed-order systems of ordinary differential operators in the self-adjoint setting, a rigorous analysis for systems of partial differential operators has been lacking so far, especially in the non-self-adjoint setting. We aim to fill in this gap in an appropriate level. Our main results include a criteria for the absence and presence of the branch due to singularity as well as its explicit description in the latter case.

9. **Florian Leben**

   Title: *A Limit-Point- and Limit-Circle Classification for $\mathcal{PT}$-symmetric operators*

   We consider a second-order differential equation

   $$-y'' + q(x)y(x) = \lambda y(x) \tag{1}$$

   with complex-valued potential q and eigenvalue parameter λ. In $\mathcal{PT}$-symmetric quantum mechanics x is on a contour $\Gamma \subset \mathbb{C}$. If the contour Γ is chosen in a very simple way, $\Gamma := \left\{xe^{i\phi \operatorname{sgn} x} : x \in \mathbb{R}\right\}$, then the above problem splits into two differential equations on the semi-axis $[0, \infty)$ and on $(-\infty, 0]$, respectively. We provide a limit-point/limit-circle-classification of this problem and a first (rough) estimate for the spectrum of the semi-axis operators. Moreover, via boundary conditions at zero, we associate with (1) a full line operator which is a one-dimensional perturbation of the direct sum of the semi-axis operators. We characterize all boundary conditions at zero such that the corresponding full line operator is selfadjoint in a Krein space or $\mathcal{PT}$-symmetric. A key result of our investigation is the following: The resolvent set of the full line operator associate to (1) is non-empty and the spectrum consists of isolated eigenvalues which accumulate to infinity only.

10. **Jiří Lipovský**

    Title: *Eigenvalue asymptotics for the damped wave equation on metric graphs*

    Linear damped wave equation on finite metric graphs is considered and its asymptotical spectral properties are researched. In the case of linear damped wave equation on an abscissa there is one high frequency abscissa, one sequence of eigenvalues with real part approaching to a constant value. In the case of a graph the location of high frequency abscissas can be determined only by the averages of the damping function on each edge of an equilateral graph. For an equilateral graph we find lower and upper bounds on the number of high frequency abscissas depending on the number of its edges and its structure.

This is a joint work with Pedro Freitas.

11. **Vladimir Lotoreichik**

    Title: *Non-self-adjoint waveguides with Robin boundary conditions*

    We will discuss the *Robin Laplacian* $-\Delta_\beta^\Omega$ on an unbounded domain $\Omega \subset \mathbb{R}^d$, $d \geq 2$, of a waveguide type with a complex coefficient $\beta\colon \partial\Omega \to \mathbb{C}$ in the boundary condition. Under reasonable assumptions on β, we show that the spectrum of $-\Delta_\beta^\Omega$ is located inside a parabolic region of the complex plane. The argument relies on subtle decay properties of the underlying (energy-dependent) *Neumann-to-Dirichlet map.*

    Particular attention will be paid to the spectrum of $-\Delta_\beta^\Omega$, when Ω and β possess certain, physically meaningful symmetries. In the mathematical language, these symmetries make the operator $-\Delta_\beta^\Omega$ self-adjoint in a properly defined *Krein space.* In this setting we are able to exclude non-real spectrum inside certain subsets of the complex plane. In order to obtain this result we employ the concept of spectra of definite type.

    This talk is based on joint works with J. Behrndt, M. Langer and J. Rohleder [1] and with P. Siegl [2].

    [1] J. Behrndt, M. Langer, V. Lotoreichik, and J. Rohleder, Decay of M-functions and spectral properties of closed extensions of symmetric operators, *in preparation.*

    [2] V. Lotoreichik and P. Siegl, Spectra of definite type in waveguide models, *Proc. Amer. Math. Soc.* **145** (2017), 1231–1246.

12. **Haruya Mizutani**

    Title: *Eigenvalue bounds for non-self-adjoint Schrödinger operators with the inverse square potential*

    Abstract: In this talk, we discuss spectral properties of the Schrödinger operator of the form $H_0 + V = -\Delta - \frac{(n-2)^2}{4|x|^2} + V$, $n \geq 3$, with complex potential V. We show Keller type estimates for individual discrete eigenvalues, which measure the radius of a disc containing the discrete spectrum in terms of the L^p norm of V. This extends the result for $-\Delta + V$ by R. L. Frank (2011) to the operator $H_0 + V$. The main new ingredient in the proof is the uniform Sobolev estimate for the resolvent of H_0. In the proof of uniform Sobolev estimates, we decompose H_0 into the radial part and otherwise so that the radial part can be estimated by uniform Sobolev estimates for the 2D free resolvent. For the latter part, we employs an abstract perturbation method to reduce the proof to that of a weighted resolvent estimate of H_0 on the orthogonal complement of the space of radial functions.

13. **Ali Mostafazadeh**

    Title: *Dynamical theory of scattering, invisible configurations of the ζe^{2iax} potential, and common zeros of Bessel functions*

    Abstract: Similarly to the S-matrix, the transfer matrix includes complete information about the scattering properties of a given real or complex potential. Dynamical theory of scattering is based on the observation that the transfer matrix can be obtained as a solution of a time-dependent Schrödinger equation for a non-self-adjoint effective Hamiltonian operator. In one dimension this describes a non-unitary two-level quantum system. This theory has the advantage of expressing the reflection and transmission amplitudes in terms of the solution of an initial-value problem for a linear second order equation defined on a unit circle. For the ζe^{2iax} potential, it offers explicit expressions for the reflection and transmission amplitudes in terms of Bessel functions. We use these to provide a complete characterization of exact unidirectionally and bidirectionally invisible configurations of this potential. The result relates an open mathematical problem on the existence of common zeros of certain pairs of Bessel functions to that of special bidirectionally invisible configurations.

    [1] A. Mostafazadeh, A Dynamical Formulation of One-Dimensional Scattering Theory and Its Applications in Optics, Ann. Phys. (NY) 341, 77-85 (2014).

    [2] A. Mostafazadeh, Dynamical Theory of Scattering, Exact Unidirectional Invisibility, and Truncated ζe^{2iax} potential, J. Phys. A: Math. Theor. 49, 445302 (2016).

14. **Ivica Nakić & Krešimir Veselić**

Title: *Perturbation of eigenvalues of Klein Gordon operators*

We give estimates for the changes of the eigenvalues of the Klein Gordon operator under the change of the potential. Included are both inclusion bounds for both standard and essential spectrum as well as two sided bounds for discrete eigenvalues. A major difference to standard selfadjoint environments lies in the fact that here the 'selfadjointness scalar product' varies with the potential.

15. **Nicolas Raymond**

Title: *Non-accretive Schrödinger operators and exponential decay of their eigenfunctions*

We consider non-self-adjoint electromagnetic Schrödinger operators on arbitrary open sets with complex scalar potentials whose real part is not necessarily bounded from below. Under a suitable sufficient condition on the electromagnetic potential, we introduce a Dirichlet realisation as a closed densely defined operator with non-empty resolvent set and show that the eigenfunctions corresponding to discrete eigenvalues satisfy an Agmon-type exponential decay.

This is a joint work with D. Krejčiřík, J. Royer and P. Siegl.

16. **Frank Rösler**

Title: *A Bound on the Pseudospectrum for a Class of Non-Normal Schrödinger Operators*

Abstract: We are concerned with the non-normal Schrödinger operator $H = -\Delta + V$ on $L^2(\mathbb{R}^n)$, where $V \in W^{1,\infty}_{\text{loc}}(\mathbb{R}^n)$ and $\operatorname{re} V(x) \geq c|x|^2 - d$ for some $c, d > 0$. The spectrum of this operator is discrete and its real part is bounded below by $-d$. In general, the ε-pseudospectrum of H will have an unbounded component for any $\varepsilon > 0$ and thus will not approximate the spectrum in a global sense.

By exploiting the fact that the semigroup e^{-tH} is immediately compact, we show a complementary result, namely that for every $\delta > 0$, $R > 0$ there exists an $\varepsilon > 0$ such that the ε-pseudospectrum

$$\sigma_\varepsilon(H) \subset \{z : \operatorname{re} z \geq R\} \cup \bigcup_{\lambda \in \sigma(H)} \{z : |z - \lambda| < \delta\}.$$

In particular, the unbounded component of the pseudospectrum escapes towards $+\infty$ as ε decreases.

Additionally, we give two examples of non-selfadjoint Schrödinger operators outside of our class and study their pseudospectra in more detail.

17. **Alexander Sakhnovich**

Title: *Weyl theory for skew- and non-self-adjoint operators and applications*

Abstract: Weyl theory is an essential part of the spectral theory of self-adjoint operators. It is successfully applied to the physically important non-self-adjoint operators as well. We consider the cases of discrete and continuous skew-self-adjoint Dirac systems, system with a rational dependence on the spectral parameter and other physically interesting examples.

18. **Stephan Schmitz**

Title: *Block Diagonalization of Unbounded Operator Matrices*

Abstract: In this talk, results on the diagonalization of off-diagonal operator/form perturbations of a diagonal operator are presented.

In the operator perturbation setting, a diagonalization of an unbounded sign-indefinite Hamiltonian that describes massless Dirac fermions in the presence of an impurity in graphene is proven.

In the case of form perturbations, a diagonalization of the Stokes block operator in Hydrodynamics is provided. In particular, it is shown that the rotation angle between the positive subspaces of the Stokes operator and its diagonal part can be bounded via the familiar (generalized) Reynolds number.

This talk is based on joint work with K. A. Makarov and A. Seelmann and recent results with L. Grubišić, V. Kostrykin, K. A. Makarov and K. Veselić.

19. **Axel Torshage**

Title: *On spectral divisors for a class of non-selfadjoint operator polynomials*

Abstract: Let H be a normal operator in the Schatten-von Neumann class S_p, with a not necessarily trivial kernel. For a compact operator K define $P_0 = (I + K)H$ and denote by $P(\cdot)$ the bounded operator polynomial

$$P(\lambda) := \sum_{i=0}^{n} \lambda^i P_i, \tag{2}$$

where P_i is compact for $i = 1, \ldots, k-1$, and $P_k = I$ for some $k \leq n$. In this talk, we present sufficient conditions for finding a spectral divisor of order k centered around the origin of the non-selfadjoint operator polynomial $P(\cdot)$. The spectral divisor is used to prove completeness and minimality of the system of eigenvectors and associated vectors of $P(\cdot)$. Moreover, we consider an application in optics that can be reduced to an operator polynomial of the form (2).

This talk is based on a joint work with Christian Engström.

20. **Carsten Trunk**

Title: *Spectrum of $\mathcal{PT}$ symmetric operators*

Abstract: A prominent class of $\mathcal{PT}$ symmetric problems is of the form

$$(\tau y)(x) := -y''(x) + x^2(ix)^\epsilon y(x), \quad \epsilon \geq 2, \text{ and } x \in \Gamma,$$

where Γ is a contour in $\mathbb{C}$ which is, in general, different from the real line and satisfies, according to the rules of $\mathcal{PT}$ symmetry, some additional conditions.

In the talk of Florian Leben it is shown how one can associate to such a problem a $\mathcal{PT}$ symmetric operator which is simultaneously selfadjoint in the Krein space $(L_2(\mathbb{R}), [\cdot,\cdot])$, where $[\cdot,\cdot]$ is given by the usual L_2–product $(\cdot,\cdot)$ via the Gramian $\mathcal{P}$, i.e. $[\cdot,\cdot] := (\mathcal{P}\,\cdot,\cdot)$. Moreover it is shown that the spectrum of such an operator consists of isolated eigenvalues only which accumulate at ∞.

In our talk we will discuss the location of the (point) spectrum of such operators and we will determine areas in the complex plane which are free of eigenvalues.

21. **Sergey N. Tumanov**

Title: *Exceptional Points and the Real Spectral Locus for Complex Airy Operator*

Abstract: We consider PT-symmetric Sturm-Liouville operators

$$T(\varepsilon) = -\frac{d^2}{dx^2} + \varepsilon P(x), \quad \varepsilon > 0,$$

in the space $L_2(-a, a)$, $0 < a \leqslant \infty$, where P is subject to the condition $P(x) = -\overline{P(-x)}$. The spectra of these operators are symmetric with respect to the real axis and discrete, provided that the interval $(-a, a)$ is finite and P is not a singular potential.

The spectrum of the operator $T(\varepsilon)$ is real for sufficiently small values of the parameter ε and in this case $T(\varepsilon)$ is similar to a self-adjoint operator.

For large values of ε the complex eigenvalues do appear and the number of non-real eigenvalues increases as $\varepsilon \to \infty$.

We'll talk about one example — complex Airy operator: $P(x) = ix$, $a = 1$ and analyze the dynamics of real eigenvalues.

The real spectral locus

$$E = \Big\{(\varepsilon, \lambda) \;\Big|\; \varepsilon > 0,\ \lambda \in \sigma(T(\varepsilon)) \cap \mathbb{R}\Big\}$$

consists of the infinite number of disjoint unbounded regular analytic curves in $\mathbb{R}^2$.

With increase of ε from 0 to $+\infty$ complex eigenvalues arise only in the collision of pairs of real eigenvalues in exceptional points ε_k, $k \in \mathbb{N}$: $\varepsilon_k = |\alpha_k\sqrt{3}/2|^3$, where $\alpha_k \neq 0$ — zeros of $\mathrm{Bi}(z) - \sqrt{3}\,\mathrm{Ai}(z)$ lying on the ray $\arg z = \pi/3$ numbered in ascending absolute value.

Exceptional points correspond to $(\varepsilon_k, 1/\sqrt{3})$ points on the locus E. In other words pairs of real eigenvalues collide only in $\lambda = 1/\sqrt{3}$.

The talk is based on the joint papers with A. A. Shkalikov.

22. **Oktay Veliev**

    Title: *On the spectral analysis of the Schrödinger operator with a periodic PT-symmetric potential*

    Abstract: I am going to give a talk about the Schrödinger operator $L(q)$ with a complex-valued PT-symmetric periodic potential q. A basic mathematical question of PT-symmetric quantum mechanics concerns the reality of the spectrum of the considered Hamiltonian. First we consider the general spectral property of the spectrum of $L(q)$ and prove that the main part of its spectrum is real and contains the large part of $[0, \infty)$. Using this we find necessary and sufficient condition on the potential for finiteness of the number of the nonreal arcs in the spectrum of $L(q)$. Moreover, we find necessary and sufficient conditions for the equality of the spectrum of $L(q)$ to the half line and consider the connections between spectrality of $L(q)$ and the reality of its spectrum for some class of PT-symmetric periodic potentials. Finally we give a complete description, provided with a mathematical proof, of the shape of the spectrum of the Hill operator with optical potential $4\cos^2 2x + 4iV \sin 2x$.

23. **Joe Viola**

    Title: *Understanding the Schrödinger evolution via the Hamilton flow for a (non-self-adjoint) degree-2 polynomial Hamiltonian*

    Abstract: Let $p(x, \xi)$ be a complex-valued polynomial on $\mathbb{R}^{2n}$ obeying a certain weak ellipticity condition. We discuss the relationship between the Hamilton flow $\exp H_p$ and the Schrödinger evolution $\exp(-iP)$ for P the Weyl quantization of p. Following and extending the ideas of Hörmander and Howe for the quadratic case, we compute the L^2 operator norm of $\exp(-iP)$ in geometric terms and obtain a composition law which applies to a broad class of integral operators with nondegenerate Gaussian kernels. The proofs use an extended definition of the Schrödinger evolution via an FBI-Bargmann reduction based on works of J. Sjöstrand and established in joint work with A. Aleman.

24. **Monika Winklmeier**

    Title: *Spectral decomposition of linear operators*

    Abstract: In this talk we consider a linear operator S whose resolvent set contains the imaginary axis. If the resolvent is uniformly bounded along the imaginary axis, then there exist invariant subspaces such that the spectrum of S restricted to them belongs either to left or to the right complex half plane. This talk is based on a joint work with Christian Wyss.

25. **Ian Wood**

    Title: *Construction of the selfadjoint dilation of a maximal dissipative operator*

    Abstract: We introduce an abstract framework for a maximally dissipative operator A and its anti-dissipative adjoint. This is similar to the way that boundary triples naturally associate boundary operators with an adjoint pair of operators, allowing e.g. Weyl functions to be introduced in an abstract setting, In the current framework, we construct the selfadjoint dilation of A using the Straus characteristic function. The advantage of this construction is that the parameters arising in the dilation are explicitly given in terms of parameters of A (such as coefficients of a differential expression). The abstract theory will be illustrated by the example of dissipative Schrödinger operators.

26. **Christian Wyss**

    Title: *Dichotomy of Hamiltonian operator matrices from systems theory*

    Abstract: In systems theory, the Hamiltonian operator matrix is a block operator matrix of the form

$$T = \begin{pmatrix} A & -BB^* \\ -C^*C & -A^* \end{pmatrix}$$

where A, B, C are linear operators on Hilbert spaces. It is closely connected to the operator Riccati equation

$$A^*X + XA - XBB^*X + C^*C = 0,$$

whose solutions X are in one-to-one correspondence with invariant graph subspaces of T. To obtain such subspaces, we use a dichotomy property of the operator matrix T: Under appropriate conditions, the imaginary axis belongs to the resolvent set and there exist invariant subspaces corresponding to the spectrum in the right and left half-plane, respectively. One can then use the symmetry of T in an indefinite inner product to show that these subspaces are in fact graph subspaces.